Ingram's
Mensuration,
by
Trotter.

A
COMPLETE SYSTEM
OF
MENSURATION;

ADAPTED TO THE

Use of Schools, Private Students, and Practical Men:

COMPREHENDING

LOGARITHMIC ARITHMETIC, PRACTICAL GEOMETRY, PLANE TRIGONOMETRY, MENSURATION OF SURFACES AND OF SOLIDS, OF CONIC SECTIONS AND OF THEIR SOLIDS, SURVEYING, MENSURATION OF HEIGHTS AND DISTANCES, LEVELLING, MENSURATION OF ARTIFICERS' WORKS, ETC.

BY ALEXANDER INGRAM,

Author of A Concise System of Mathematics, The Principles of Arithmetic, &c.

WITH MANY IMPORTANT ADDITIONS AND IMPROVEMENTS,

BY JAMES TROTTER,

Of the Scottish Naval and Military Academy; Author of A Manual of Logarithms, A Key to Ingram's Mathematics, &c.

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⁂ The references throughout the Work are to the complete edition of Ingram's Concise System of Mathematics.

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LOGARITHMS.

DEFINITION AND NOTATION OF LOGARITHMS.

LOGARITHMS are a set of artificial numbers invented and formed into Tables for the purpose of facilitating arithmetical computations. They are adapted to the natural numbers in such a manner, that by their aid Addition supplies the place of Multiplication, Subtraction that of Division, Multiplication that of Involution, and Division that of Evolution or the Extraction of Roots.

Let a series of Numbers in Arithmetical Progression be adapted to another in Geometrical Progression, so that the least term of the one may correspond with the least term of the other, and the rest in order; thus,—

Arithmetical Progression,	0,	1,	2,	3,	4,	5,	6,	7,	&c.
Geometrical Progression,	1,	4,	16,	64,	256,	1024,	4096,	16384,	&c.

Now, let it be required to multiply together any two terms of the Geometrical series, as 16 and 64. This may be done by adding 2 and 3, the corresponding terms of the Arithmetical series, for the sum 5 is the term corresponding to 1024, the product. Again, if we wish to divide any term of the Geometrical series by any other, as 1024 by 16, we have only to subtract 2 from 5, the corresponding terms of the Arithmetical series, for the difference 3 is the term corresponding to the quotient 64.

Hence, the use of such an adaptation is manifest; but it is very limited in the present state of the series. We may, however, easily extend it by interpolating an Arithmetical mean between every two terms of the Arithmetical series, and a Geometrical mean between every two terms of the Geometrical series, when the number of terms will be doubled; thus,—

Arith. Prog.	0,	0·5,	1,	1·5,	2,	2·5,	3,	3·5,	4,	4·5,	5,	5·5,
Geom. Prog.	1,	2,	4,	8,	16,	32,	64,	128,	256,	512,	1024,	2048,

6,	6·5,	7,	7·5,	&c.
4096,	8192,	16384,	32768,	&c.

These progressions may again be interpolated in the same manner by new terms, and the process carried on continually, till at length every integer shall occur in the Geometrical series, or a number so near to it, that the difference may be neglected without sensible error, and then the numbers in the Arithmetical series corresponding to these integers may be called their Logarithms.

In forming the common Tables of Logarithms, the progressions first assumed were,—

Arith. Prog.	0,	1,	2,	3,	4,	5,	6,	&c.
Geom. Prog.	1,	10,	100,	1000,	10,000,	100,000,	1,000,000,	&c.

New terms were continually interpolated, in the manner shown in the former series, until the natural numbers 1, 2, 3, 4, &c. occurred in the Geometrical series; and then the numbers in the Arithmetical series, corresponding to these natural numbers, were taken to compose the Tables of Logarithms.

In this system of Logarithms it is manifest that the Logarithm of any number and that of another, 10, 100, 1000, &c. times greater or less will consist of the same decimal fraction called the *mantissa*, and differ only in the integral part; so that all numbers, whether they are integers, decimals, or partly integral and partly decimal, have the same positive quantity for the decimal part of their logarithm; thus,—

The logarithm of	1839	is 3·264582	
......	183·9	is 2·264582	
......	18·39	is 1·264582	
......	1·839	is 0·264582	
......	·1839	is $\bar{1}$·264582*	or 9·264582
......	·01839	is $\bar{2}$·264582	or 8·264582
......	·001839	is $\bar{3}$·264582	or 7·264582

APPLICATION OF LOGARITHMS.†

The index or integral part of the logarithm of any whole or mixed number, is always *one less* than the number of integral figures of which the number consists; and in decimal fractions, the index which is negative is that number which points out *the distance* of the first significant figure from the place of units. Instead of negative indices, their *arithmetical complements* to 10 are often used. Thus, if there is no cipher between the decimal point and the first significant figure of the decimal, the index is $\bar{1}$ or 9; if there is one cipher be-

* When the index or characteristic of the log. is negative, the sign — is generally put above it, in order to distinguish the index from the *mantissa*, which must always be considered + or affirmative.

† The Problems and Rules given under this head apply only to Tables of Logarithms of numbers from 1 to 10,000, such tables being of more common use than those which are more extensive.

tween them, the index is $\bar{2}$ or 8; if two ciphers are between them, it is $\bar{3}$ or 7, and so on.

The indices being thus readily found are generally omitted in the common logarithmic tables, and the mantissa only of the logarithm is inserted.

PROBLEM I. To find the logarithm of any given number from the table.

1. If the given number is not greater than 100, the logarithm, with its index prefixed, will, in ordinary tables, be found in the first page, immediately opposite the number; thus, the log. of 65 is 1·812913, and that of 88 is 1·944483.

2. If the number consists of three figures, it will be found in the margin on the left hand side of the page, and the decimal part of its log. immediately opposite, in the column under 0; thus, the log. of 536 is 2·729165, and that of 760 is 2·880814.

3. If the number consists of four figures, the first three will be found in the margin on the left hand side of the page, and immediately opposite, and under the fourth figure found at the top of the page, will be got the mantissa of the logarithm; thus, the log. of 7846 is 3·894648, and that of 37·56 is 1·574726.

4. If the given number consists of more than four figures, find the logarithm of the first four figures, as before directed; multiply the difference between this logarithm and the next higher in the table by the remaining figures of the given number, and cut off as many figures from the product as are in the multiplier; the remaining figures added to the logarithm of the first four figures will give the logarithm required.

NOTE.—The mean difference given under D, in the right hand column, may be used, except in the first three pages of the table, where they vary rapidly.

To find the logarithm of 476·384. Opposite the number 476 in the margin, and under 3 at the top, is ·677881, the difference in the column D, on the right hand side of the page, is 91, which, multiplied by 84, and two figures cut off, gives 76 to be added to ·677881, and since there are three integral figures the index is 2, the logarithm of 476·384 is therefore 2·677957. Again, to find the logarithm of 1056·472. The logarithm of the first four figures is ·023664, the difference between this and the next higher in the table is 411 (that in the column marked D being 412). Now, 411 × 472, and three figures cut off, gives 194, which added to the logarithm of the first four figures, and the proper index prefixed, gives 3·023858 as the required logarithm. If the number had been 105647·2, the decimal part of the logarithm would have been the same, but the index would have been 5.

As examples for practice, find the logarithms of

4719	Ans. 3·673850
47218	Ans. 4·674108
210·394	Ans. 2·323034
7218·65	Ans. 3·858456
21056·3	Ans. 4·323382
714219·5	Ans. 5·853939
·0009146	Ans. $\bar{4}$·961231 or 6·961231

PROBLEM II. To find the number corresponding to a given logarithm.

If the given number be found in the table, the first three figures of the number will be found on the same line in the left hand column of the page, and the fourth at the top or bottom of the column in which the logarithm is found. But if the logarithm be not found exactly in the table, take the number answering the next less, subtract this logarithm from the given one, and also from the next greater in the table; and annexing ciphers to the first remainder, divide it by the other, or by the mean difference found in the column marked D, to find the fifth, sixth, &c. figures of the number. The integral places must always be one more than the units in the index of the given logarithm, and the rest decimals; thus,—

To find the number answering to the logarithm 3·721906. This logarithm is not found in the table, but the next less is ·721893, and the number answering to this is 5271; now the difference between the given logarithm and that next less found in the table is 13, and the difference between that found in the table and the next greater is 82 (that in the column marked D is also 82); hence, 1300 ÷ 82 = 16; the number therefore corresponding to log. 3·721906 is 5271·16.

Find the numbers corresponding to the following logarithms:—

Log. 2·754096	Ans. 567·67
Log. 4·069375	Ans. 11732·08
Log. 3·721986	Ans. 5272·13
Log. $\bar{2}$·364875	Ans. ·0231672
Log. $\bar{4}$·300847	Ans. ·000199915
Log. $\bar{1}$·219634	Ans. ·1658187

PROBLEM III. To find the arithmetical complement of a logarithm.

Subtract the log. from 10, an integer, or subtract the right hand figure from 10 and all the rest from 9; thus the arithmetical complement of 2·730459 is 7·269541, and of $\bar{3}$·826464 is 12·173563.

Problem IV. To perform Multiplication by logarithms.

Add the logarithms of the factors; the sum is the logarithm of the product; thus,—

To multiply 7825 by 873 we have—

Log.	7825	=	3·893484
Log.	873	=	2·941014
Log.	6831218·8	=	6·834498 their sum.

The correct product is 6831225, or 6·2 greater than that found by logarithms; but when there are various operations, the final error is scarcely appreciable, as the slight inaccuracy of one operation generally balances that of another.

Note.—A negative index must be subtracted when the logarithm is added, and added when the logarithm is subtracted; thus,—

To multiply 786 by ·0073.

Log.	786	=	2·895423	or	2·895423
Log.	·0073	=	$\bar{3}$·863323	or	7·863323
Log.	5·7378	=	0·758746		0·758746

When the positive index is used, in adding we reject 10 from the index, but in subtracting, we borrow 10.

Multiply 78·36 by 8·5		Ans. 666·06
. . . .	210·4 by ·00372	Ans. ·7826875
. . . .	486·95 by 2·0087	Ans. 978·1364
. . . .	21896 by 274·35	Ans. 6007166·7

Problem V. To divide one number by another.

Subtract the log. of the divisor from that of the dividend; the remainder is the log. of the quotient; thus,—

To divide 78634 by 27 we have

Log.	78634	=	4·895610
Log.	27	=	1·431364
Log.	2912·37	=	3·464246

The quotient is true to the last place of decimals; again—
To divide 7856 by ·0053, we have—

Log.	7856	=	3·895201	or	3·895201
Log.	·0053	=	$\bar{3}$·724276	or	7·724276
Log.	1482263	=	6·170925		6·1 925

Divide 274·15 by 3·5	Ans. 78·3285
.... 48160 by 27·6	Ans. 1744·928
.... 57·486 by 389·5	Ans. ·147589
.... 2816·4 by ·0038	Ans. 741158

Problem VI. To involve a number by logarithms.

Multiply the log. of the given number by the name of the power; the product is the log. of the power; thus,—

To raise 27 and ·005 to the 3d power—

$$\begin{array}{llll} \text{Log. } 27 = 1{\cdot}431364 & \text{log. } {\cdot}005 = \bar{3}{\cdot}698970 & \text{or} & 7{\cdot}698970 \\ \qquad\qquad\quad 3 & \qquad\qquad\qquad 3 & & \qquad 3 \\ \hline \text{Log. } 19683 = 4{\cdot}294092 & \text{L. } {\cdot}000000125 = \bar{7}{\cdot}096910 & & 3{\cdot}096910 \end{array}$$

Note.—After multiplying the negative index, the carriage from the decimal part of the logarithm must be subtracted from the product. If the positive index is used, 10 times the name of the power lessened by *one*, must be taken from the product.

Raise ·0437 to the 4th power.	Ans. ·0000036469
... 32 to the 3d power.	Ans. 32768
... ·009 to the 3d power.	Ans. ·000000729
... 2756 to the 2d power.	Ans. 7595526·3

very nearly true, the correct answer being 7595536.

Problem VII. To find any root of a number by logarithms.

Divide the logarithm of the number by the exponent of the root; the quotient is the logarithm of the root.

Note.—If the given number is a decimal, and the positive index used, prefix the name of the root lessened by *one* to the index, before dividing. If the negative index is used, add to it the least number that will make the sum divisible by the name of the root; the quotient is the index of the root; but in dividing the decimal part of the logarithm, the number added to the index must be considered as the index; thus,—

To find the 3d root of 19683 and of ·000000125.

$$\begin{array}{llll} \text{Log. } 19683 = 3)4{\cdot}294092 & \text{log. } {\cdot}000000125 = 3)\bar{7}{\cdot}096910 & \text{or} & 3)\overline{3{\cdot}096910} \\ \text{3d root } 27 = \quad 1{\cdot}431364 & \text{3 root } {\cdot}005 \quad = \bar{3}{\cdot}698970 & & 7{\cdot}698970 \end{array}$$

Find the 4th root of ·00130321	Ans. ·19
.... 3d .. of 9261	Ans. 21
.... 4th .. of 7	Ans. 1·62657
.... 3d .. of ·041063625	Ans. ·345

PRACTICAL GEOMETRY.

DEFINITIONS.

1. GEOMETRY treats of magnitude or continued quantity, and of its relation to number.

2. A SOLID is that which has three dimensions, length, breadth, and thickness.

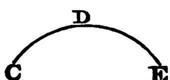
3. A SURFACE, or SUPERFICIES, is the boundary of a solid, and has only length and breadth.

4. A LINE is the boundary of a surface, and has only length.

5. A POINT is the extremity of a line. It has position, but not magnitude, as A.

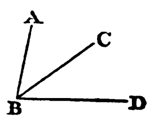
6. A STRAIGHT LINE is one every part of which points in the same direction, as AB.

7. A CURVE changes continually its direction, or it has unlike sides, a concave and a convex, as CDE.

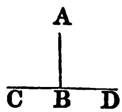


8. An ANGLE is the measure of the relative position of two straight lines which meet, or it is their inclination to one another.

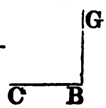
NOTE. An angle is denoted by three letters, of which the second is at the point where the lines meet, and the other two are upon the containing lines, one on each. Thus the uppermost angle is named ABC, the other CBD, and the whole angle ABD.



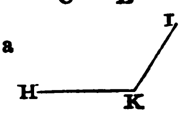
9. A straight line is said to be PERPENDICULAR to another, when it does not incline towards one end more than towards the other. Thus AB is perpendicular to CD.

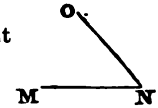


10. A RIGHT ANGLE is that made by a perpendicular, as CBG.



11. An OBTUSE ANGLE is greater than a right angle, as HKI.

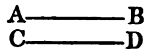




12. An Acute Angle is less than a right angle, as MNO.

13. A Plane is a surface with which a straight line will coincide, when drawn between any two points in it.

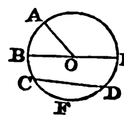
14. Parallels are straight lines in a plane, which never meet, though extended ever so far both ways, as AB and CD.



15. A Circle is a figure contained by a curve ABD, called the *circumference*, which is equally distant from a point O within it, called the *centre*.

16. The Radius AO is a straight line, drawn from the centre to the circumference.

17. The Diameter BE is a straight line, drawn through the centre O, and terminated both ways at the circumference.



18. A Chord CD is a straight line joining any two points of the circumference.

19. An Arc BCD is any part of the circumference.

20. A Semicircle is a portion of the circle cut off by a diameter, as BAE.

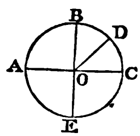
21. A Segment is a portion CFD, cut off by a chord CD.

22. A Sector is a part cut off by two radii, as AOB.

Note 1. If the radii contain a right angle, the sector is called a *Quadrant;* and, if half a right angle, it is called an *Octant.*

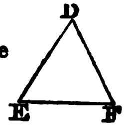
Note 2. The circumference of every circle is supposed to be divided into 360 equal parts, called *degrees*, and a degree into 60 equal parts, called *minutes*, and a minute into 60 *seconds*, and so on. Degrees are marked by a small circle at the top of the right-hand figure, minutes with one accent, seconds with two accents, &c.; thus 29° 12′ 45″ denote 29 degrees, 12 minutes, and 45 seconds.

Note 3. If two diameters AC, BE, are perpendicular to one another, they divide both the circle and the circumference into four equal parts, and form four right angles at the centre; and if the arc CB of one of these parts be divided into 90 degrees, and radii drawn to the points of division, they will divide the right angle BOC into 90 equal angles, each of which is said to be an angle of one degree, and any angle AOD at the centre is said to consist of as many degrees as the arc AD upon which it stands. The arc AD is called the *measure* of the angle AOD. Hence a right angle AOB contains 90°, an obtuse angle AOD more, and an acute angle COD less than 90 degrees.

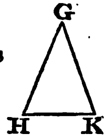


23. A Triangle is a figure contained by three straight lines.

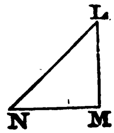
24. An EQUILATERAL TRIANGLE has its three sides equal, as DEF.



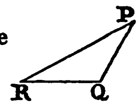
25. An ISOSCELES TRIANGLE has two of its sides equal, as GHK.



26. A RIGHT-ANGLED TRIANGLE has one right angle, as LMN. The side LN opposite the right angle is called the *Hypotenuse,* the sides NM and LM about the right angle are called the *Base* and *Perpendicular,* or the *Legs* of the Triangle.



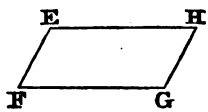
27. An OBTUSE-ANGLED TRIANGLE has one obtuse angle, as PQR.



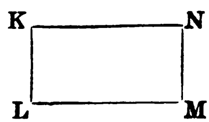
28. All others are called ACUTE-ANGLED TRIANGLES.

29. A QUADRILATERAL is a figure bounded by four straight lines.

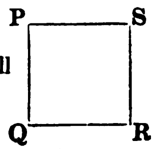
30. A PARALLELOGRAM is a quadrilateral, of which the opposite sides are parallel, as EFGH.



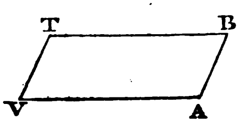
31. A RECTANGLE is a parallelogram which has right angles, as KLMN.



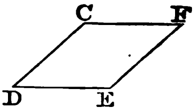
32. A SQUARE is a rectangle which has all its sides equal, as PQRS.



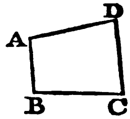
33. A RHOMBOID is a parallelogram which has no right angles, as TVAB.



34. A Rhombus is a rhomboid which has all its sides equal, as CDEF.

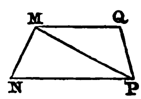


35. A Trapeze, or Trapezium, is a quadrilateral which has not its opposite sides equal, as ABCD.

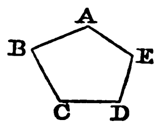


36. A Trapezoid has two sides parallel, but not the other two, as MNPQ.

37. A Diagonal is a straight line, which joins two opposite angles of a figure, as MP.

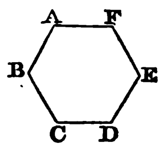


38. A Polygon is a figure contained by more than four straight lines, as ABCDE.



39. A Polygon of five sides is called a *Pentagon;* one of six sides, a *Hexagon;* of seven sides, a *Heptagon;* of eight sides, an *Octagon;* of nine sides, a *Nonagon;* of ten sides, a *Decagon,* &c.

40. A Regular Polygon is that which has all its sides and all its angles equal, as ABCDEF.



41. An Irregular Polygon is that which has not all its sides and all its angles equal.

42. Similar Figures are such as have all the angles of the one equal to all the angles of the other, and the corresponding sides about the angles of each proportional.

43. The Perimeter of a figure is the sum of all its sides.

44. A Proposition is a general term, either implying something to be demonstrated, or some operation to be performed. In the former case it is called a *Theorem,* and in the latter a *Problem.*

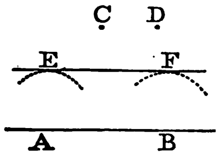
45. A Corollary is some property which obviously results from the demonstration of a proposition.

46. A Scholium is a remark or observation upon what precedes it.

PROBLEMS.

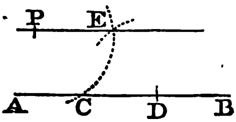
Prob. I. To draw a straight line parallel to AB, and as far from it as the point C is from D.

With the distance CD for a radius, describe arcs E and F from the centres A and B, and draw the straight line EF to touch these arcs without cutting them.

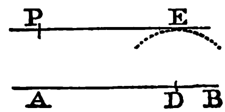


Prob. II. To draw a parallel to AB through the point P.

From P, with any sufficient radius, describe an arc cutting AB in C. Lay the radius on AB from C to D, and from D cut the arc again in E, and draw PE.

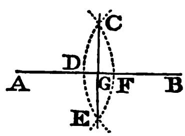


Or, with the nearest distance of P from AB for a radius, describe an arc E, from D, taken as far as possible from P, and draw a line from P to touch the arc E.



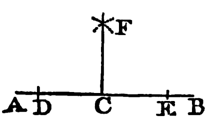
Prob. III. To bisect a given straight line AB.

With a radius greater than half the line, describe from B the arc CDE, and from A the arc CFE, cutting the former in C and E. Draw CE cutting AB in G.



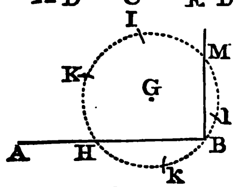
Prob. IV. To raise a perpendicular to AB at a given point in it, as C.

With any radius, from C, cut AB in D and E; and with a greater radius describe arcs from D and E, cutting one another in F, and draw CF.



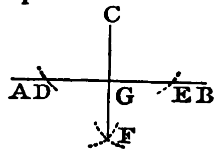
If the perpendicular is to be raised at B, the end of AB,

Place one foot at G, above AB, and extending the other to B, describe a circle cutting AB in H; then lay the radius on the circumference, from H to K, from K to I, and from I to M, and draw BM.



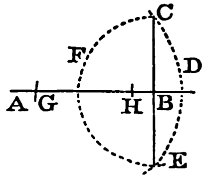
Or a straight line through H and G will give M.

Prob. V. To let fall a perpendicular upon AB from the point C above it.



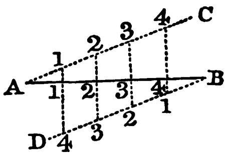
With a sufficient radius from C cut AB in D and E, and from these points describe arcs on the other side of AB, cutting one another in F, and draw CF, cutting AB in G.

If the point C be above the end of AB,



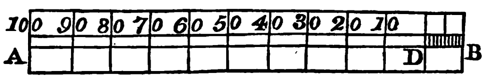
From any point G in AB, with the radius GC, describe the arc CDE; and from any other point H, in AB, with the radius HC, describe the arc CFE, cutting the former in E, and draw CE.

Prob. VI. To divide a straight line AB into any number of equal parts, suppose five.



Through A and B draw any parallels AC and BD, on different sides of AB. Take any convenient distance, and lay it four times (one less than the given number) from A on AC, and from B on BD; then join the first on AC to the fourth on BD, the second on AC to the third on BD, and so on in order, and the joining lines will divide AB into five equal parts.

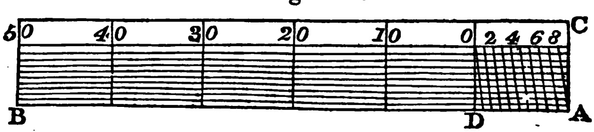
Prob. VII. To make a plain scale, or one of equal parts.



Draw any straight line AB, and take any convenient distance, and lay it eleven times from A to B, and divide the last one BD into 10 equal parts; then each of the large divisions will be 10, and each of the small divisions 1.

For a scale of feet and inches, divide BD into 12 equal parts; then each of the large divisions will be a foot, and each of the small ones an inch.

Prob. VIII. To make a diagonal scale.

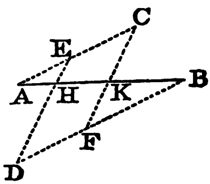


Having drawn AB, and divided it as in the plane scale,

draw AC perpendicular to AB, and on it lay any small distance 10 times, and through the points of division draw parallels to AB, and through the great divisions of AB draw parallels to AC; divide AD and C0 each into 10 equal parts, and draw a line from 0 to the first division of AD, and from the first division of 0C to the second of AD, and so on.

To take from this scale a number consisting of three figures, as 546, call one of the large divisions 100, or take 5 of them, call one of the divisions on 0C 10, or take 4 of them, and for the units reckon one parallel on the diagonal for each unit; or count 6 parallels on the diagonal through 4, and bring the foot on the large 5, along that division to the sixth parallel.

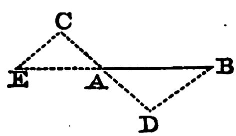
PROB. IX. To divide a straight line AB in any proportions, as of 3, 5, 7.



Draw any parallels AC and BD, through A and B on different sides of AB. From any scale of equal parts take the extent from 0 to 3, and lay it on AC, from A to E. Take 7 from the same scale, and lay it on BD, from B to F; then take 5, and lay it from E to C, and from F to D; and join ED, CF, cutting AB in H and K. AH : HK : : 3 : 5, and HK : KB : : 5 : 7.

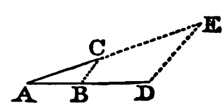
NOTE. In the same way, AB may be divided similarly to a given divided line.

PROB. X. To produce a straight line AB, so that the whole shall be to the produced part in a given ratio, as of 5 to 2.



Through A draw any straight line DAC, lay 2 from A to C, and 5 from C to D towards A. Join BD, and parallel to it draw CE. Then BE : EA : : 5 : 2.

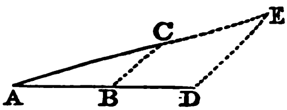
PROB. XI. To find a third line proportional to two given straight lines, as 4 and 6.



Make any angle BAC, and lay the first term 4 from A to B, and the second term 6 both from A to C and from B to D. Join BC, and draw DE parallel to it. Then CE = 9 is the third proportional.

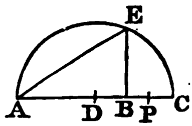
Prob. XII. To find a fourth line proportional to three given ones, as 8, 6, and 12.

Make any angle BAC. Lay the first 8 from A to B, the second 6 from B to D, and the third 12 from A to C. Join BC, and draw DE parallel to it. Then CE is the fourth proportional.



Prob. XIII. To find a mean proportional between two straight lines, as 9 and 4.

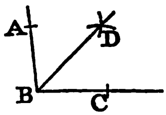
On the same straight line lay AB 9 and BC 4, and bisect AC in D; then with the radius DA describe the semicircle AEC, and draw BE perpendicular to AC. It is the mean proportional, for AB : BE : : BE : BC.



Note. Make AP = AE, then AP or AE is a mean proportional between AC and AB; therefore AC : AB : : $AC^2 : AP^2$.

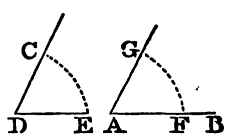
Prob. XIV. To bisect a given angle ABC.

From B, with any radius, cut the sides in A and C. From A describe the arc D, and from C cut it in D, and join BD, the angle ABD = CBD.



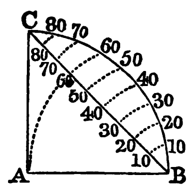
Prob. XV. To make, at A in AB, an angle equal to the angle CDE.

From D, with any radius, cut DC, DE, in C, E; and from A, with the same radius, describe the arc FG. Take the extent from C to E, and lay it on the arc from F to G, and draw AG, the angle FAG = CDE.



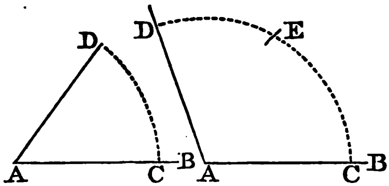
Prob. XVI. To make a scale of chords.

Draw AC perpendicular to AB. From A, with any radius, describe the arc BC, and let it be divided into 90 equal parts, (it is here divided into 9,) and draw BC; and, with one foot in B, transfer the extents to each of the divisions, from the arc to BC. Then BC is a line of chords.



Note. The radius AB is equal to the chord of 60°.

Prob. XVII. To make an angle of any number of degrees, at A in AB.

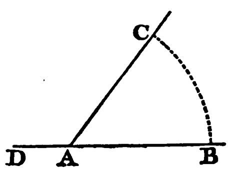


Take 60° from the line of chords, and from A describe an arc, cutting AB in C.

If the given angle do not exceed 90°, as 54°, take it from the line of chords, and lay it on the arc from C to D; draw AD, then BAD is the angle required.

If the given angle be greater than 90°, as 112°, take a less number from the chords; lay it from C to E, lay the rest from E to D, and draw AD; then BAD is the angle required.

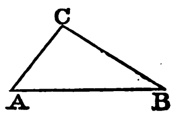
PROB. XVIII. To measure a given angle BAC.



With the chord of 60°, from A describe the arc BC. Lay BC on the line of chords, and it will show the number of degrees in the angle BAC.

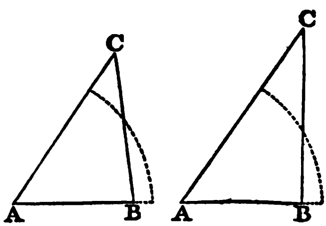
If the extent from B to C be greater than the line of chords, measure part of the arc, and then the rest, and add them. Or produce BA to D, and measure CAD, which, subtracted from 180°, leaves BAC.

PROB. XIX. To make a triangle, of which the three sides are given, viz. 186, 257, and 324 feet.



Draw a straight line AB. Take 324 from the diagonal scale, and lay that extent from A to B. Take 186 from the scale, and from A describe an arc; then with 257 for a radius, from B cut that arc in C, and join AC, CB.

PROB. XX. To make a triangle, of which two sides and an angle are given, viz. 256, 384, and 54° 40′.

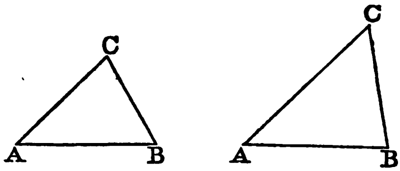


Make the angle BAC 54° 40′, and make AB 256; then, if the given angle be between the given sides, make AC 384, and join BC.

But if one of the sides be opposite to the given angle, with 384 for a radius, from B cut AC in C, and join BC.

NOTE. If it had been required to make AB 384, and BC 256, the problem would have been impossible; because 256 for a radius would not reach from B to AC. If BC were 340, it would be perpendicular to AC. If BC were greater than 340, but less than 384, it would cut AC in two points, so that two different triangles could then be made with the same things given.

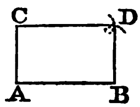
PROB. XXI. To make a triangle, of which two angles 43° 36′, and 57° 44′, and one side 297 feet, are given.



Make the angle BAC 43° 36′, and make AB 297. Then, if the other given angle is to be adjacent to the given side, make ABC 57° 44′; but if it is to be opposite to the given side, add the given angles, and subtract the sum 101° 20′ from 180°. The remainder 78° 40′ is the angle ABC, and then ACB is 57° 44′.

NOTE. If in either of these problems a right angle is given, it is to be made 90°, or a perpendicular is to be drawn.

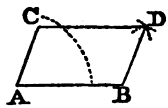
PROB. XXII. To make a rectangle, of which the sides are given; suppose 428 and 246 feet.



Draw AC perpendicular to AB; make AB 428, and AC 246 feet; then with 246 for a radius, from B describe the arc D; and with 428 for a radius, from C cut that arc in D, then join BD, CD.

NOTE. If AC be made equal to AB, the figure will be a square.

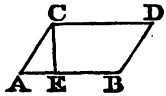
PROB. XXIII. To make a parallelogram, of which two sides, 436 and 243 feet, and an angle, 67° 30′, are given.



Make the angle BAC 67° 30′; make AB 436, and AC 243; then with 243 for a radius from B describe the arc D, and with 436 from C cut that arc in D, then draw CD, BD.

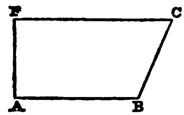
PROB. XXIV. To make a parallelogram, of which there are given two sides 421 and 234 feet, and the perpendicular upon one of them, suppose the longest, from the end of the other 196.

Draw CD parallel to AB, at the distance of 196 feet from it; and with 234 for a radius from A cut CD in C, and make AB, CD each 421; then join AC, BD, and let fall the perpendicular CE.



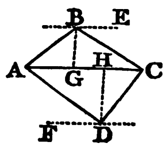
PROB. XXV. To make a quadrilateral, of which all the sides, 256, 348, 436, and 297 feet, and an angle contained by the two first, 87° 44′, are given.

Make the angle BAF 87° 44′; make AB 256, and AF 348; then from F, with 436 for a radius, describe an arc, and with 297 from B cut that arc in C, and draw FC, CB.



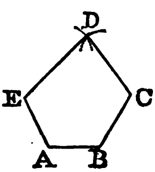
PROB. XXVI. To make a quadrilateral, of which are given two sides 268 and 394, the diagonal from their intersection 473, and the perpendiculars upon it from their extremities 188 and 234 feet.

Make AC 473, and draw parallels to it on different sides at the distances of 188 and 234, as BE, DF. With 268 for a radius from A cut BE in B, and with 394 cut DF in D. Join AB, BC, CD, DA, and let fall the perpendiculars BG, DH, on AC.



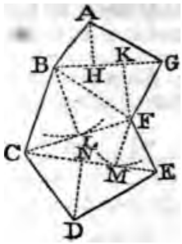
PROB. XXVII. To make a pentagon of which all the sides are given, 236, 194, 253, 318, and 372 feet; and two angles, suppose those at the extremities of the second side, 112° and 124°.

Make AB 194 feet; at A make the angle BAE 112°, and at B the angle ABC 124°; make AE 236, and BC 253; then with 318 for a radius from C describe the arc D, and from E with 372 cut it in D, and draw CD, ED.



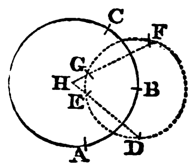
NOTE. In like manner may any polygon be made, of which all the sides are given, and all the angles except three.

PROB. XXVIII. Given two sides of a figure 234 and 348, the diagonals 438, 385, 452, and 537, and the perpendiculars upon the diagonals from the angles 183, 248, 315, 212, and 274; to construct the figure.

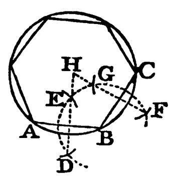


First, by Prob. XXVI., make the quadrilateral ABFG, of which AB is 234, BG 438, BF 385, AH 183, and FK 248. From B with the radius 315 describe an arc, and from F draw FC to touch it; make FC 452, and join BC. From F with 212 describe an arc, draw CE to touch it, and make CE 537. Draw a parallel to CE at the distance of 274 from it, and from C with 348 cut the parallel in D, and join CD, DE, EF, and draw the perpendiculars BL, FM, DN.

Prob. XXIX. To describe a circle that shall pass through three given points, A, B, C, not in a straight line.



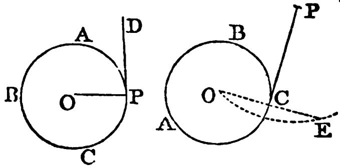
With a radius greater than half the distance of B from A or C describe a circle about B; with the same radius from A cut the circle in D and E, then from C cut it in F and G. Join DE, FG, meeting one another in H; it is the centre, from which the circle described through A shall pass through B and C.



Note 1. If ABC be a triangle, a circle may be described about it by this problem. And in the same way, by taking three points in the circumference, or in any arc of a circle, the centre of that circle may be found.

Note 2. The circumference which passes through three of the angular points of a regular polygon passes through all the rest; and therefore a circle may be described about it, or inscribed within it, by this problem.

Prob. XXX. To draw a straight line from a given point P, to touch a given circle ABC.

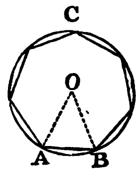


If P be in the circumference, draw PO to the centre, and PD perpendicular to it.

If P be without the circle; from P describe the arc OE through the centre O, and from O, with the diameter of ABC for a radius, cut the arc in E;

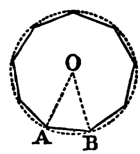
then draw EO, meeting the circumference in C. Join PC, and it will touch the circle.

PROB. XXXI. To make a regular polygon of a given number of sides in a given circle ABC.



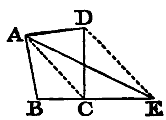
Divide 360° by the number of sides; the quotient is the angle at the centre subtended by one of them. Draw a radius AO, and make the angle AOB equal to the quotient. Join AB, and place straight lines all around the circle equal to AB, and they will form the polygon required.

PROB. XXXII. To make a regular polygon of a given number of sides, upon a given straight line, as AB 365 feet.



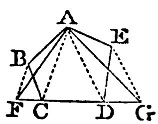
Divide 360° by twice the number of sides, and subtract the quotient from 90°. At A and B make the angles BAO, ABO, each equal to the remainder, and the point O in which the sides meet is the centre of the circle containing the polygon. From O describe a circle through A, and place lines equal to AB all round in it.

PROB. XXXIII. To make a triangle equal to a given quadrilateral ABCD.



Draw the diagonal AC, and parallel to it, through D, draw DE, meeting BC produced, if necessary, in E, and join AE; then the triangle ABE is equal to the quadrilateral ABCD. For the triangle ACE = ACD.

PROB. XXXIV. To make a triangle equal to a given pentagon ABCDE.

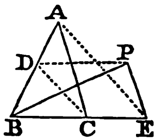


Join AC, and draw BF parallel to it, meeting CD in F, then join AF, and the triangle AFC = ABC; and thus the pentagon is reduced to the quadrilateral AFDE. Let this be reduced as before to the triangle AFG, then AFG = ABCDE.

NOTE. In the same way may any polygon be reduced to a triangle, only the number of operations will increase with the number of the sides of the figure.

PROB. XXXV. To reduce a triangle ABC to another,

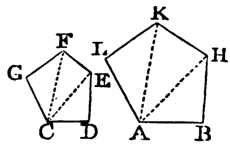
which shall have its base in the same straight line with that of the given triangle, and its vertex at a given point P.



Draw PD parallel to BC, meeting AB in D. Join DC, and through A draw AE parallel to DC, and join PB, PE. If DE were joined, the triangle ADC = EDC, and ABC = DBE = PBE.

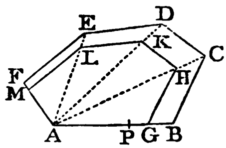
NOTE. By this and the preceding problem, any polygon may be reduced to a triangle, which shall have its vertex at a given point.

PROB. XXXVI. To construct a figure upon a given straight line AB, which shall be similar to a given figure CDEFG.



Join CE, CF, to reduce the given figure to triangles. At A make the angle BAH = DCE, HAK = ECF, and KAL = FCG. Also at B make the angle ABH = CDE; at H make AHK = CEF; and at K make AKL = CFG. Then ABHKL is similar to CDEFG.

PROB. XXXVII. To construct a figure which shall be similar to a given figure ABCDEF, and have a given ratio to it, as that of 7 to 9.

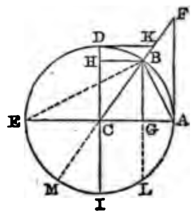


As 9 is to 7, so make AB to AP, and find AG, a mean proportional between AB, AP, by Prob. XIII. Having drawn the diagonals AC, AD, AE, draw GH parallel to BC, meeting AC in H, draw HK parallel to CD, KL parallel to DE, and LM to EF; then the figure AGHKLM is similar to ABCDEF, and has to it the ratio of 7 to 9.

PLANE TRIGONOMETRY.

TRIGONOMETRY is the method of determining the sides and angles of triangles, and of expressing them in known measures. This is done by means of the ratios which certain straight lines in and about the circle have to its radius.

DEFINITIONS.



1. The SINE BG of an arc AB, is a straight line drawn from B, one of its extremities, perpendicular to the diameter AE, which passes through the other.

2. The VERSED SINE AG of an arc AB, is that portion of the diameter AE upon which the sine is perpendicular, intercepted between the sine and the arc.

3. The TANGENT AF of an arc AB is a perpendicular to the radius CA at one extremity of the arc, and meets at F the diameter MB, which passes through the other extremity B.

4. The SECANT CF of an arc AB, is a straight line drawn from C the centre, to F the farthest extremity of the tangent.

5. The sine, versed sine, tangent, and secant of an arc AB, are called the sine, versed sine, tangent, and secant of the angle ACB measured by that arc to the radius AC.

6. The COMPLEMENT of an arc AB, or angle ACB, is what it wants of a quadrant or 90°. Thus BD or BCD is the complement of AB or ACB.

7. The SUPPLEMENT of an arc AB, or of an angle ACB, is what it wants of 180°. Thus BE or AM is the supplement of AB, and BCE or ACM the supplement of ACB.

Cor. 1. An arc or angle, and its supplement, have the same sine, tangent, and secant; for BG is the sine of BE or BCE, AF the tangent of AM or ACM, and CF the secant of AM or ACM.

Cor. 2. The versed sine EG of BCE (or the supplemental versed sine of ACB), together with AG the versed sine of ACB, is equal to the diameter AE.

8. What the arc wants of the whole circumference, or the angle of four right angles, is sometimes called the *explement:* Thus BDEMLA is the explement of AB, or of ACB.

9. The sine, versed sine, tangent, and secant of the complement of an arc or angle, are called the cosine, coversed sine, cotangent, and cosecant of the arc or angle. Thus BH or CG is the cosine of AB or ACB, DH is its coversed sine, DK its cotangent, and CK its cosecant.

Cor. 1. The cosine CG, together with the versed sine AG, is equal to the radius AC.

Cor. 2. The radius is equal to the sine or versed sine of 90°, and to the tangent or cotangent of 45°.

NOTE 1. We generally write sin. for sine, cos. for cosine, tan. for tangent, sec. for secant, ver. for versed sine, cov. for coversed sine, suv. for supplemental versed sine, cot. for cotangent, cosec. for cosecant, cho. for chord, R. or rad. for radius, and D. or dia. for diameter.

From these definitions the equations which express the values of the trigonometrical lines in terms of each other are easily derived.

1. Since the diameter which bisects an arc, bisects also the chord at right angles, it follows that half the chord of any arc is equal to the sine of half that arc: Thus $BG = \frac{1}{2}BL$.

2. In the right-angled triangle CGB, $CB^2 = CG^2 + GB^2$, or the square of the radius is equal to the sum of the squares of the sine and cosine of any arc; hence $\text{sin.} = \sqrt{(R^2 - \text{cos.}^2)}$, $\text{cos.} = \sqrt{(R^2 - \text{sin.}^2)}$, or if radius $= 1$, then $\text{sin.} = \sqrt{(1 - \text{cos.}^2)}$, and $\text{cos.} = \sqrt{(1 - \text{sin.}^2)}$.

The triangles CGB, CAF, CDK, being evidently similar, we have

3. $CG : GB :: CA : AF$, or the cosine of an arc is to its sine as the radius to the tangent; therefore $\text{tan.} = \frac{R \times \text{sin.}}{\text{cos.}} = \frac{\text{sin.}}{\text{cos.}}$, if radius $= 1$.

4. $GB : CG :: CD : DK$, or the sine of an arc is to its cosine as the radius is to the cotangent; hence $\text{cot.} = \frac{R \times \text{cos.}}{\text{sin.}} = \frac{\text{cos.}}{\text{sin.}}$, if radius $= 1$.

5. $CG : CB$ or $CA :: CA : CF$, or the radius is a mean

proportional between the cosine of an arc and its secant; whence sec. $= \frac{R^2}{\text{cos.}} = \frac{1}{\text{cos.}}$, if radius $= 1$.

6. GB : CB or CD :: CD : CK, or the radius is a mean proportional between the sine of an arc and its cosecant; therefore cosec. $= \frac{R^2}{\text{sin.}} = \frac{1}{\text{sin.}}$, if radius $= 1$.

7. AF : CA or CD : : CD : DK, or the radius is a mean proportional between the tangent of an arc and its cotangent; hence tan. $= \frac{R^2}{\text{cot.}} = \frac{1}{\text{cot.}}$, if radius $= 1$, and cot. $= \frac{R^2}{\text{tan.}} = \frac{1}{\text{tan.}}$, if radius $= 1$.

8. The triangle CAF being right angled, $CF^2 = CA^2 + AF^2$, or the square of the secant is equal to the sum of the squares of the radius and the tangent; hence sec. $= \sqrt{(R^2 + \text{tan.}^2)} = \sqrt{(1 + \text{tan.}^2)}$, if radius $= 1$, and tan. $= \sqrt{(\text{sec.}^2 - R^2)} = \sqrt{(\text{sec.}^2 - 1)}$, if radius $= 1$.

9. In the right-angled triangle CDK, $CK^2 = CD^2 + DK^2$, or the square of the cosecant is equal to the sum of the squares of the radius and the cotangent; therefore cosec. $= \sqrt{(R^2 + \text{cot.}^2)} = \sqrt{(1 + \text{cot.}^2)}$, if radius $= 1$, and cot. $= \sqrt{(\text{cosec.}^2 - R^2)} = \sqrt{(\text{cosec.}^2 - 1)}$, if radius $= 1$.

10. From the similar triangles EGB, BGA, EG : GB : : GB : GA, or the sine of an arc is a mean proportional between the versed sine and its supplemental versed sine; that is, between the versed sine and the sum of the radius and cosine; therefore vers. $= \frac{\text{sin.}^2}{R + \text{cos.}} = \frac{\text{sin.}^2}{1 + \text{cos.}}$, if radius $= 1$.

11. Since $CG^2 = BH^2 = DH \times HI$ or $DH \times (CD + CH)$ it is obvious that the cosine of an arc is a mean proportional between the sum and the difference of the radius and the sine, or between the coversed sine and the sum of the radius and sine; hence cov. $= \frac{\text{cos.}^2}{R + \text{sin.}} = \frac{\text{cos.}^2}{1 + \text{sin.}}$, if radius $= 1$.

OF THE SIGNS OF THE TRIGONOMETRICAL LINES.

Geometrical quantities, when expressed analytically, are estimated from some given point or line, and are considered as + or —, according as they lie on the one or on the other side of that point or line.

The sines are estimated from the diameter EA, and the cosines from the centre C; and if we consider the sines as *positive* when they lie above the diameter, and the cosines when they lie on the right-hand side of the centre, it is obvious, that, in the first quadrant AD, the sines and cosines are both positive. In the second quadrant DE, the sine ly-

ing still above the diameter is *positive*, but the cosine having changed its position in regard to the centre is now *negative*. The sine changing its position in the third quadrant EI, is now set off below the diameter, and the cosine remaining as in the second quadrant, they are therefore both *negative*. And, in the fourth quadrant, the sine still lying below the diameter is *negative*, while the cosine having resumed its original position in regard to the centre is *positive*.

The signs of the other quantities may be easily determined from the preceding equations, for since tan. $= \frac{\text{sin.}}{\text{cos.}}$, it follows, that when the signs of the sine and cosine are *alike*, that of the tangent is positive, and when they are *unlike*, the sign of the tangent is negative.

The following table exhibits the mutations of the signs of the different quantities for each quadrant of the circle :—

Quadrants.	Sin.	Cos.	Tan.	Cot.	Sec.	Cosec.	Vers.	Cov.
1.	+	+	+	+	+	+	+	+
2.	+	—	—	—	—	+	+	+
3.	—	—	+	+	—	—	+	+
4.	—	+	—	—	+	—	+	+

NOTE 1. The signs of the sine and cosecant, of the cosine and secant, and of the tangent and cotangent, are respectively *alike*; and the signs of the versed and coversed sines are always *positive*, the former being always set off from A in the same direction, and the latter from E in the contrary direction.

NOTE 2. The sines, cosines, &c. may be considered not only as belonging to arcs less than four quadrants, but also to those arcs increased by any number of complete circumferences.

OF THE CONSTRUCTION OF A TABLE OF SINES, COSINES, &C.

Various methods may be employed for computing the numerical values of the sines, cosines, &c., but we shall only exhibit Sir Isaac Newton's method.

If x be any arc of a circle, whose radius is unity, it was shown by Newton that (See Appendix)

$$\text{Sin. } x = x - \frac{x^3}{1.2.3} + \frac{x^5}{1.2.3.4.5} - \frac{x^7}{1.2.3.4.5.6.7} + \text{\&c., and}$$

$$\text{Cos. } x = 1 - \frac{x^2}{1.2} + \frac{x^4}{1.2.3.4} - \frac{x^6}{1.2.3.4.5.6} + \text{\&c.}$$

Now by means of these series, and the ratio between the diameter and circumference of the circle, the sines and cosines of any arc may be found.

When the radius is unity, half the circumference is 3·14159265358979З, &c., and as there are 180° or 10800′ in

a semicircle, it is obvious that, if we divide the former by the latter, we will obtain the length of an arc of 1 minute $= \cdot 00029088821$; whence, if the arc is 1 minute,

$$
\begin{aligned}
x &= \cdot 00029088821 \\
-\tfrac{1}{6}x^3 &= -\cdot 0000000000004 \\
\therefore \text{Sin. } x &= \cdot 0002908882 = \text{the sine of 1 minute.}
\end{aligned}
$$

$$
\begin{aligned}
\text{Again from } & 1\cdot 0000000000 \\
\text{Take } \tfrac{1}{2}x^2 = & 0\cdot 0000000423 \\
\therefore \text{Cos. } x = & \cdot 9999999577 = \text{the cosine of 1 minute.}
\end{aligned}
$$

Let the arc be 5°, to find its sine and cosine.

Here $\frac{5 \times 3\cdot 14159265}{180} = \cdot 08726646 = x =$ the length of an arc of 5°; hence

$$
\begin{aligned}
x &= 0\cdot 08726646 \\
-\tfrac{1}{6}x^3 &= -0\cdot 00011076 \\
+\tfrac{1}{120}x^5 &= +0\cdot 00000004 \\
\therefore \text{Collecting, sin. } x &= 0\cdot 08715574 = \text{the sine of 5 degrees.}
\end{aligned}
$$

$$
\begin{aligned}
\text{And for the cosine } & 1\cdot 00000000 \\
-\tfrac{1}{2}x^2 &= -0\cdot 00380771 \\
+\tfrac{1}{24}x^4 &= +0\cdot 00000241 \\
\therefore \text{Collecting, cos. } x &= 0\cdot 99619470 = \text{the cosine of 5 degrees.}
\end{aligned}
$$

This method may be employed for the sines and cosines at the beginning and end of the quadrant, for when the arc does not exceed 10′, the first two terms of the series give the sine and cosine true to 15 places; and when it does not exceed 1°, the first three terms give them true to the same number of places, but the nearer the arc is to 45°, the more slowly do these series converge; and therefore the greater are the number of the terms that must be employed in the calculation.

NOTE. It is necessary to compute the sines only, as the cosines are more easily found from the equation, $\text{cos.} = \sqrt{(1 - \text{sin.}^2)}$. p. 24.

By this method the sines and cosines may be computed as far as 45°; and it is obvious, from the definitions, that the sines and cosines will also be found from 45° to 90°, for sine 50° = cosine 40°, and cosine 60° = sine 30°, &c.

The tangents and secants may be readily obtained by Formulæ III. and V., pages 24 and 25, when the cotangents and cosecants will also be known.

The versed sines are $= 1 \mp \text{cos.}$, according as the arc is greater or less than 90°, and the coversed sines are the complements of the versed sines to 1.

The sines, cosines, &c. which we have been computing are called Natural Sines, Cosines, &c., and when these are arranged in a table from 1′ up to 90°, they form what is termed the *Trigonometrical Canon.*

If the logarithms of all the natural sines, cosines, &c. be taken from the common logarithmic tables, and 10 added to their indices, these will form the tables of logarithmic sines, cosines, &c.

The logarithmic sines, &c. are supposed to be computed to the radius 10,000,000,000, in order that the smallest arc likely to be used in calculation, may not have a negative index; but the natural sines, &c. are computed to the radius 1, hence the reason of adding 10 to the indices.

OF THE TABLES OF SINES, TANGENTS, &C.

The common tables have the degrees at the top, and the minutes on the left-hand side, when the degrees are less than 45°; but if greater, the degrees are marked at the bottom, and the minutes on the right-hand side.

1. Required the logarithmic sine of 37° 23′ 12″.

Turn to the page which has 37° at the top, and come down the column titled *Sine* at the top, to the line that has 23′ on the left-hand side, and you will find 9·783292, the sine of 37° 23′; and the difference between it and the sine of 37° 24′ is 166. Then as 60″ is to 12″, so is 166 to 33, the proportional difference for 12″, which, added to 9·783292, gives 9·783325, the logarithmic sine of 37° 23′ 12″.

2. Required the degrees and parts of a degree of which 10·273846 is the logarithmic tangent.

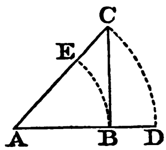
Look for the nearest tangent 10·273716, and because it is titled *Tang.* at the bottom, take the degrees at the foot, and the minutes on the right-hand side, where are found 61° 58′. The difference between this tangent and the one above it is 305, and the difference between it and the given one is 130; therefore 305 : 130 : : 60″ : 26″, so that 10·273846 is the tangent of 61° 58′ 26″.

The log. secant is found by subtracting the log. cosine from 20, and the log. cosecant is found by subtracting the log. sine from 20.

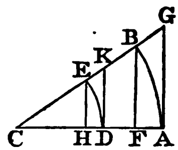
3. Required the nat. sine of 57° 26′ 20″. Ans. ·842818.
4. . . . . log. cosine of 67° 31′ 40″. 9·582331.
5. . . . . log. secant of 73° 27′ 45″. 10·545700.
6. Nat. cosine is ·747682, what is the arc? 41° 36′ 36″.
7. Log. secant is 10·475546, what is the arc? 70° 27′ 19″.

SOLUTION OF RIGHT-ANGLED TRIANGLES.

THE first thing to be done in resolving right-angled triangles is to make one of the sides the radius of a circle, the centre of which is at an acute angle, and thus to determine what the other sides would be in that circle.



If from the centre A, with the radius AC, the arc CD be described, then BC will be the sine of CAB, and AB its cosine. But if the centre be at C, and the circle pass through A, then AB is the sine of C, and BC its cosine. Hence when the hypotenuse is radius, the other sides are the sines of their opposite angles, or the cosines of their adjacent angles. Again, if from the centre A, with the radius AB, the arc BE be described, then BC is the tangent of A, and AC is its secant.



Suppose ACB any angle, and AB an arc described with the radius of the circle, from which the sines, tangents, &c. in the tables were calculated; then BF is the sine in the tables, CF the cosine, AG the tangent, and CG the secant in the tables. Let CEH be a right-angled triangle. If CE be radius EH will be the sine of C, and CH its cosine. Hence CE : EH : : CB : BF (Theor. 1. Trig.); that is, CE is to EH as the radius of the tables is to the sine of C in the tables. In like manner CE is to CH as the radius is to the cosine of C in the tables (Theor. 1. Trig., Cor. 1.) In the same way if CDK is the triangle, and CD the radius, CD is to DK as the radius is to the tangent of C in the tables (Theor. 1. Trig.), and DC is to CK as the radius is to the secant of C in the tables (Theor. 1. Trig., Cor. 2.); so that after determining the names of the sides of the triangle, any two sides are to one another as their names in the tables.

The terms of the proportion, however, must be so arranged, that the thing required shall be the last term, thus:

To find EH, R : sin. C : : CE : EH

To find CE, sin. C : R : : HE : EC

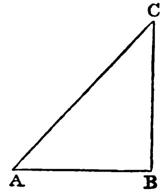
To find C, CE : EH : : R : sin. C.

And these three are all the variations which are requisite. But the student should accustom himself to state them without hesitation. Before proceeding to the numerical solution, he should also construct the triangle geometrically, as di-

rected in Problems XVII. to XXI. PRACTICAL GEOMETRY, distinguishing the given sides by a dash across them, and the given angles by one or two dots.

1. In the triangle ABC, right angled at B, are given the hypotenuse AC 324 feet, and the angle BAC 48° 17′; to find the base AB, and perpendicular BC.

If AC be radius, and A the centre, CB is the sine of A, and AB its cosine. Wherefore,



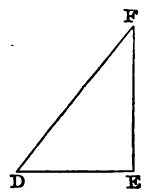
R : sin. A :: AC : CB, and R : cos. A :: AC : AB.

Sin. A 48° 17′	log.	9·872998	cos. A	log.	9·823114
AC 324	log.	2·510545	AC 324	log.	2·510545
Sum		12·383543	Sum		12·333659
Radius		10·000000	Rad.		10·000000
CB 241·85	log.	2·383543	AB 215·6	log.	2·333659

NOTE. Instead of subtracting the logarithm of the first term from the sum of the logarithms of the second and third, it is preferable to take the arithmetical complement of the first and add the three together.

2. Given DE 1254 feet, and the angle D 51° 19′; to find the hypotenuse DF, and the perpendicular EF.

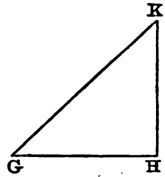
DE being radius, EF is the tangent and DF the secant of D.



R : tan. D :: DE : EF.			R : sec. D* :: DE : DF.		
Tan. D 51° 19′ — R.		0·096545	Sec. D 51° 19′ — R.		0·204109
DE 1254	log.	3·098298	DE 1254	log.	3·098298
EF 1566·18	log.	3·194843	DF 2006·35	log.	3·302407

3. Given the angle G 43° 38′, and the opposite side HK 186 feet; to find the hypotenuse GK, and the base GH.

This may be wrought as the last by first finding GKH. Or, GK being radius, KH is the sin. of G; and GH being radius, HK is the tan. of G.



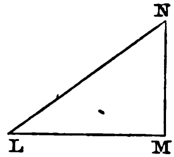
* *The log. secant is readily found by subtracting the log. cosine from 20.*

Sin. G : R : : HK : KG, and tan. G : R : : KH : HG.

HK 186 + R.	log. 12·269513	HK + R.	log. 12·269513
Sin. G 43° 38′	log. 9·838875	tan. G	log. 9·979274
GK 269·549	log. 2·430638	GH 195·09	log. 2·290239

4. Given the hypotenuse LN 415 inches, and the perpendicular MN 249; to find the angles, and LM.

LN being radius, NM is the sine and LM the cosine of L; whence



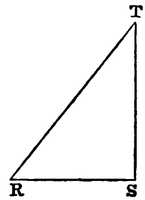
LN : NM : : R : sin. L. R : cos. L : : NL : LM.

NM 249 + R.	log. 12·396199	Cos. L 36° 52′ 12″ — R.	$\bar{1}$·903089
LN 415	log. 2·618048	LN 415	log. 2·618048
Sin. L 36° 52′ 12″	log. 9·778151	LM 332	log. 2·521137

NOTE. LM is equal to the square root of the product of the sum and difference of LN and NM $= \sqrt{664 \times 166} = \sqrt{110224} = 332$.

5. Given the base RS 53 miles, and the perpendicular ST 67; to find the angles, and hypotenuse RT.

If RS is made the radius, then ST is the tangent and RT the secant of R; therefore



RS : ST : : R : tan. R. R : sec. R : : SR : RT

ST 67 + R.	log. 11·826075	Sec. R 51° 39′ 16″ — R.	0·207326
RS 53	log. 1·724276	RS 53	log. 1·724276
Tan. R 51° 39′ 16″	10·101799	RT 85·4284	log. 1·931602

NOTE. The square of RT is equal to the sum of the squares of RS and ST; therefore RT $= \sqrt{53^2 + 67^2} = \sqrt{7298} = 85{\cdot}4284$.

6. Given the hypotenuse 893, and the base 586 chains.
Ans. Angle at base 48° 59′ 17″, perpendicular 673·833 ch.

7. Given the base 326 yards, and the vertical angle 64° 40′.
Ans. Hypotenuse 360·686, perpendicular 154·33 yards.

8. Given the perpendicular 286, and vertical angle 71° 24′.
Ans. Hypotenuse 896·666, base 849·8314.

9. Given the hypotenuse 963 links, and vertical angle 41° 48′. Ans. Base 641·87, perpendicular 717·892 links.

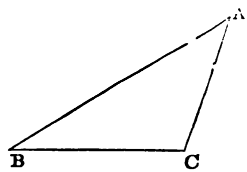
SOLUTION OF OBLIQUE-ANGLED TRIANGLES.

Case I. When two sides and an angle opposite to one of them is given.

Any two sides of a triangle are to one another as the sines of the angles opposite to them. Thus BC : CA : : sin. A : sin. B, or sin. A : sin. B : : CB : CA (Theor. 2. Trig.)

The former order is to be used when an angle is required, and the latter when a side.

1. Given two sides AB 532, and BC 358 feet, and the angle at C 107° 40′; to find the angles at A and B, and the side AC.



AB : BC : : sin. C : sin. A.

Sin. C (107° 40′) 72° 20′*		9·979019
BC 358 feet	log.	2·553883
		12·532902
BA 532	log.	2·725912
Sin. A 39° 53′		9·806990

B=180°—(C+A), and sin. C : sin. B : : BA : AC.

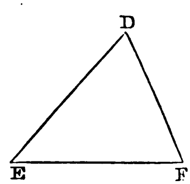
Sin. B 32° 27′		9·729621
BA 532	log.	2·725912
		12·455533
Sin. C 107° 40′		9·979019
AC 299·58	log.	2·476514

2. Given AB 232, and BC 345 yards, and the angle at C 37° 20′; to find the angles at A and B, and the side AC.

By proceeding in the same way, the angle at A may be either 64° 24′ or 115° 36′, and therefore the angle at B may be either 78° 16′ or 27° 4′, and AC 374·56 or 174·07. For AB being less than BC, there are two triangles which have each the given things in them.

3. Two places are 560 feet from one another, and at a station 258 feet from the first place, their distance subtended an angle of 63° 28′. Required the distance of the station from the other place. Ans. 625·468 feet.

4. Given two angles D 63° 48′, and E 49° 25′, and the side EF opposite to D 275 yards; to find DE and DF. The angle at F is=180°—(D+E)=66° 47′.



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* When the angle is greater than 90°, take the sine, tangent, &c. of its supplement.

Sin. D : sin. E : : EF : FD.			Sin. D : sin. F : : FE : ED.		
Sin. E 49° 25′	log.	9·880505	Sin. F 66° 47′	log.	9·963325
EF 275	log.	2·439333	EF 275	log.	2·439333
		12·319838			12·402658
Sin. D 63° 48′	log.	9·952918	Sin. D 63° 48′	log.	9·952918
FD 232·766	log.	2·366920	DE 281·67	log.	2·449740

5. Given the angles at E 49° 25′, and at F 63° 48′, and the side EF 275; to find ED and DF.

Ans. ED 268·488, and DF 227·255.

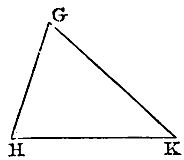
6. A ship sailing due north observes a cape bearing N. 54° 12′ W.; and after sailing 27 miles, the cape bore S. 70° 30′ W. Required her distances from it.

Ans. First distance 30·957, second distance 26·636 miles.

CASE II. When two sides and the angle between them are given.

Add and subtract the sides to get their sum and difference. Subtract the angle from 180°, and take half the remainder, to get half the sum of the unknown angles. Then as the sum of the sides is to their difference, so is the tangent of half the sum of the unknown angles to the tangent of half their difference (Theor. 4. Trig.) Having thus found the half difference, add it to the half sum to get the angle opposite to the greater side, and subtract it to get the less angle; after which the third side is found by Case I.

7. Given the sides GH 133, and HK 176 yards, and the angle at H 73° 16′; to find the angles at G and K, and the side GK.



KH+HG : KH—HG : : tan. ½(G +K) : tan. ½(G—K).			Sin. G : sin. H : : HK : KG.		
KH — HG 43	log.	1·633468	Sin. H 73° 16′		9·981209
Tan. ½(180° — H) 53° 22′		10·128679	HK 176	log.	2·245513
		11·762147			12·226722
KH + GH 309	log.	2·489958	Sin. G 63° 58′		9·953537
Tan. ½(G — K) 10° 36′	log.	9·272189	GK 187·58	log.	2·273185
Angle G		63° 58′			
Angle K		42° 46′			

8. Given GH 237, and GK 482 feet, and the angle at G 77° 48′; to find the angles at H and K, and HK.

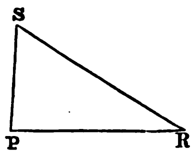
Ans. H 73° 59′ 39″, K 28° 12′ 21″, and HK 490·1135 feet.

9. Given HK 78, and KG 168, and the angle K 128° 26′.
Ans. H 35° 48′ 20″, G 15° 45′ 40″, HG 224·943.

CASE III. When the three sides are given.

Add the three sides, and from half the sum subtract the side opposite to the angle sought; then take the arithmetical complements of the logs. of the two sides containing the angle sought, and the logarithms of the half sum and of the remainder, and add these four together, and half the sum will be the log. cosine of half the angle sought. (Theor. 5. Trig. Cor.)*

10. Given the sides SP 230, PR 365, and SR 426 feet; to find the angles.



SP	230	ar. co. log.	7·638272
PR	365	ar. co. log.	7·437707
SR	426		
½)1021			
½ Sum	510·5	log.	2·707996
	426		
Rem.	84·5	log.	1·926857
			½)19·710832
½P 44° 12′ 24″		cosine	9·855416
P 88° 24′ 48			

In the same manner the angle S is found to be 58° 55′ 25″.

11. Given the sides SP 1248, PR 728, and RS 956 feet.
Ans. The angle R 94° 40′ 50″, P 49° 46′ 16″.

12. Given SP 375, PR 275, and RS 196.
Ans. The angle S 45° 17′ 26″, P 30° 25′ 58″.

PROMISCUOUS EXERCISES.

1. Given the hypotenuse of a right-angled triangle 528 feet, and one of the acute angles 39° 27′.
Ans. The opposite side 335·493, adjacent side 407·7104 feet.

2. Given the base 256, and the adjacent angle 57° 28′.
Ans. Hypotenuse 476·022, perpendicular 401·324 feet.

3. Given the perpendicular 297 feet, and the angle at the base 36° 48′.
Ans. Hypotenuse 495·806, base 397·0073 feet.

---

* Let $s =$ half the sum of the three sides, then $SP \times PR : s \times (s - SR) :: \text{rad.}^2 : \cos.^2 \tfrac{1}{2} P$, or $\frac{\text{rad.}^2}{SP \times PR} \times s \times (s - SR = \cos.^2 \tfrac{1}{2} P = 2 \log. R - (\log. SP + \log. PR) + \log. s + \log. (s - SR) = 2 \log. \cos. \tfrac{1}{2} P =$ ar. co. log. SP + ar. co. log. PR + log. s + log. $(s - SR) = 2 \log. \cos. \tfrac{1}{2} P$, which is the rule.

4. Given the hypotenuse 1268, and perpendicular 428 yards.

Ans. The base 1193·583, adjacent angle 19° 43′ 37·3″.

5. Given the base 674, and the perpendicular 438 yards.

Ans. Hypotenuse 803·8166 yards, angle at base 33° 1′ 4·4″.

6. Given the hypotenuse 97, and the base 38 miles.

Ans. Perpendicular 89·247 miles, angle at base 66° 56′ 11″.

7. Given the base 326, and the vertical angle 67° 30′.

Ans. The hypotenuse 352·86, perpendicular 135·034.

8. In an oblique triangle, given two angles 46° 48′ and 114° 26′, and the side opposite the lesser 254 feet.

Ans. Other sides 317·2328, and 112·0974 feet.

9. Given two angles 56° 24′ and 74° 28′, and the side between them 354. Ans. Other sides 451·0104, and 389·898.

10. Given two sides 572 and 748, and the angle opposite to the greater 67° 30′.

Ans. Angle opposite less 44° 57′ 1·5″, third side 748·269.

11. Given two sides 356 and 294, and the angle opposite to the lesser 51° 27′.

Ans. Other angles 71° 15′ 39″ and 57° 17′ 21″, or 108° 44′ 21″ and 19° 48′ 39″; third side 316·309 or 127·4079.

12. Given two sides 1864 and 1235, and included angle 73° 38′.

Ans. Other angles 68° 21′ 15·48″ and 38° 0′ 44·52″, third side 1924·155.

13. Given two sides 436 and 219, and included angle 127°.

Ans. Other angles 35° 52′ 45·72″ and 17° 7′ 14·28″, third side 594·125.

14. Given the three sides 456, 327, and 184 yards.

Ans. Angles 123° 55′ 10·8″, 36° 31′ 5·72″, and 19° 33′ 43·48″.

15. Given the sides 2586, 1482, and 1234.

Ans. Angles 144° 14′ 52·6″, 19° 33′ 49″, and 16° 11′ 18·4″.

16. Given two angles 57° 12′ and 24° 45′, and the side between them 365 poles.

Ans. Other sides 154·33, and 309·86 poles.

17. Given two sides 120 and 112 feet, and the angle opposite the less 57° 27′.

Ans. Angle opposite the greater 64° 34′ 22″ or 115° 25′ 38″, and third side 112·65 or 16·47 feet.

MENSURATION OF SURFACES.

THE area or surface of a figure is the number of square inches, feet, yards, &c. which it contains.

A square constructed upon a straight line, of which the length is an inch, is called a *square inch;* and the same is to be understood of a square foot, &c. This is called the *measuring unit,* and the area of any figure is computed by the number of those squares which it contains.

TABLE OF LINEAL MEASURES.

Inches.	Feet.				
12	1	Yards.*			
36	3	1	Poles.		
198	$16\frac{1}{2}$	$5\frac{1}{2}$	1	Furlongs.	
7920	660	220	40	1	Mile.
63360	5280	1760	320	8	1

TABLE OF SQUARE MEASURES.

Square In.	Square Feet.				
144	1	Sqr. Yds.			
1296	9	1	Sqr. Pls.		
39204	$272\frac{1}{4}$	$30\frac{1}{4}$	1	Roods.	
1568160	10890	1210	40	1	Acre.
6272640	43560	4840	160	4	1

NOTE. The acre contains 10 square chains, each 16 perches, or 100,000 square links. The chain is 66 feet in length, and is divided into 100 links, each 7·92 inches.

* The imperial yard is the distance between the centres of the points in the gold studs fixed in the brass rod belonging to the House of Commons, and titled "Standard Yard, 1760." When used, the brass must be at the temperature of 62 degrees of Fahrenheit's thermometer.

The length of a pendulum vibrating seconds of mean time, at the level of the sea, in the latitude of London, contains 39·1393 imperial inches.

SCOTCH LAND MEASURE.

Ells.	Falls.		
36	1	Roods.	
1440	40	1	Acre.
5760	160	4	1

NOTE. A Scotch ell = 37·0598 imperial inches. The Scotch chain is 74·1196 imperial feet, and consequently the Scotch acre is = 1·26118345 imperial acre.

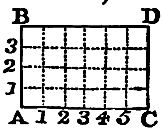
PARALLELOGRAMS.

PROB. I. To measure a right-angled parallelogram.

RULE. Multiply one of the sides by the other.

That is, AC × AB = the area (El. Geom. 15. Schol.)

1. Required the area of the rectangle ABDC, of which the sides are AB 4 yards, and AC 6.*



6
4
—
Area 24 square yards.

NOTE. If AC be divided into 6 equal parts or yards, and AB into 4, and lines be drawn parallel to the sides, the rectangle will be divided into 24 squares, each of them a square yard.

2. Required the area of a square, each side 37 feet.
Ans. 1369 square feet.

3. Required the area of a rectangle, the sides 326 and 153 feet. Ans. 49878 sq. feet = 1 acre 23 per. $6\frac{1}{4}$ yds.

4. Required the area of a square, each side 3525 links.
Ans. 124·25625 ac. = 124 ac. 1 ro. 1 per.

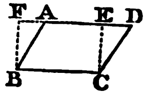
5. A rectangular space, 68 feet 3 inches long by 56 feet 8 inches broad, is to be paved with stones each 2 feet 3 inches, by 10 inches. Required how many stones it will take, and what will be the expense at 2s. 3d. for a square yard.
Ans. $2062\frac{2}{3}$ stones, expense £48, 6s. $10\frac{1}{2}$d.

PROB. II. To measure any parallelogram.

RULE. Multiply one of the sides by the perpendicular let fall upon it from the opposite side.

That is, BC × FB = the area (El. Geom. 15. Schol.)

* The student should always construct the figures upon his slate before he begins his computations.

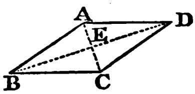


1. Required the area of the parallelogram ABCD, of which the sides are AB 214, and BC 354, and the perpendicular CE 192 feet.

$$\begin{array}{r} 354 \\ 192 \\ \hline 9|67968 \text{ square feet.} \\ \hline 4840)7552 \text{ square yards.} \\ \hline \end{array}$$

Ans. 1 acre 2 roods 9 perches $19\frac{3}{4}$ yards.

2. Required the area of a rhombus, the side 358, and the perpendicular on it 194 feet. Ans. 69452 feet.



3. Required the area of a rhombus, of which the diagonals are AC 436, and BD 623 yards.

NOTE. AC and BD bisect one another at right angles. For in the triangles AED, CED, the side AE = CE (El. Geom. 29.) AD = CD (El. Geom., Def. 34.), and ED common; whence these triangles are equal in every respect, and the angle AED = CED, or each is a right angle (El. Geom. 5.)

Ans. $623 \times 218 = 135814$ yards, = 28 ac. 9 per. $21\frac{3}{4}$ yds.

4. Required the area of a rhomboid, the sides 1234 and 762, and the perpendicular on the former 658 links.

Ans. 8·11972 ac. = 8 ac. 19 per. 4 yds. $6\frac{1}{4}$ feet.

5. Required the area of a parallelogram, the sides 56 feet 8 inches and 42 feet 10 inches, and the perpendicular on the latter 47 feet 3 inches. Ans. 2023 feet $10\frac{1}{2}$ inches.

6. Required the area of a rhomboid, the sides 24 and 18 poles, and the perpendicular upon the latter 96 yards.

Ans. 9504 sq. yds. = 1 acre 3 roods 34 per. $5\frac{1}{2}$ yards.

7. Required the area of a rhombus, the diagonals $6\frac{1}{2}$ feet and $3\frac{1}{4}$ feet. Ans. 10 feet 81 inches.

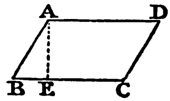
PROB. III. Given two sides and an angle of a parallelogram; to find the area.

RULE. Multiply the product of the two sides by the natural sine of the angle.

That is, BC × BA × sin. B = the area.*

Or add the logarithms of the sides and the logarithm sine of the angle: the sum, after taking 10 from the index, will be the logarithm of the area.

* The area, by Prob. 2., is BC × AE, but rad. 1 : sin. B : : BA : AE = BA × sin. B; hence BC × BA × sin. B = the area.



1. Required the area of the rhomboid ABCD, of which the sides are AB 278, and BC 456 feet, and the angle B 58° 46′.

$$
\begin{array}{r}
\text{Sin. } 58^\circ 46' = \cdot 85506 \\
456 \\
\hline
389\cdot 90736 \\
278 \\
\hline
43560)108394\cdot 24608 \text{ square feet.}
\end{array}
$$

Ans. 2 acres 1 rood 38 perches $4\frac{1}{3}$ yards.

2. Required the area of a rhombus, the side 172 ells, and an angle 72° 30′. Ans. 28214·74 ells.

3. Required the area of a rhomboid, the sides 136 and 97 yards, and the angle 73° 16′.

Ans. 12633·4 sq. yds. = 2 ac. 2 ro. 17 per. 19 yds. 1·35 ft.

4. Required the area of a rhomboid, the sides 628 and 425 links, and the angle 126°.

Ans. 2·159267 ac. = 2 ac. 25 per. 14 yds. 5·4 feet.

5. Required the area of a rhombus, the side 57 poles, and the angle 67° 45′.

Ans. 3007·08 per. = 18 ac. 3 ro. 7·08 per.

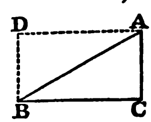
6. Required the area of a rhombus, the side 157 inches, and the angle 29° 12′. Ans. 12025·26 sq. in. = 83 ft. $73\frac{1}{4}$ in.

TRIANGLES.

PROB. IV. Given the base and the perpendicular of a triangle; to find the area.

RULE. Multiply the base by the perpendicular, and half the product will be the area.

That is, $\frac{1}{2}(BC \times AC)$ = the area (El. Geom. 15. Schol.)



1. Required the area of the right-angled triangle ABC, of which the sides about the right angle are BC 254, and AC 136 yards.

$$
\begin{array}{r}
254 \\
68 \\
\hline
4840)17272 \text{ square yards.}
\end{array}
$$

Ans. 3 acres 2 roods 10 perches $29\frac{1}{2}$ yards.

2. Required the area of a triangle ABC, the base CB 396, the side AB 278, and the perpendicular AE 174 feet.

Ans. $396 \times 87 = 34452$ square feet, = 3 ro. 6 per. $16\frac{1}{2}$ yds.

3. Required the area of a triangle, one angle 43°, adjacent side 296, and perpendicular on it 176 yards.

Ans. 26048 sq. yards, = 5 ac. 1 ro. 21 per. $2\frac{3}{4}$ yds.

4. Required the area of a triangle, the sides 156 and 97 poles, and the perpendicular upon the latter 102 poles.

Ans. 4947 perches, = 30 acres 3 roods 27 perches.

5. Required the area of a triangle, the side 684 links, the angle adjacent 137°, and the perpendicular 928 links.

Ans. 3·17376 acres, = 3 acres 27 perches $24\frac{1}{4}$ yards.

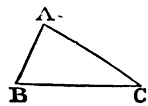
PROB. V. Given two sides and the included angle of a triangle; to find the area.

RULE. Multiply one side by half of the other, and by the natural sine of the included angle.

That is, $\frac{1}{2}AB \times BC \times \sin. B =$ the area.*

Or add the logarithms of one side and of half the other, and the logarithm sine of the angle: the sum, rejecting 10 in the index, is the logarithm of the area.

1. Required the area of the triangle ABC, of which the side AB is 534, and BC 872 links, and the angle B 63° 40′.



$$\begin{array}{r}
\text{Sin. } 63^\circ\ 40' = \cdot 89623 \\
872 \\
\hline
781\cdot 51256 \\
267 \\
\hline
100000)208663\cdot 85352 \text{ square links.} \\
\hline
2\cdot 0866385
\end{array}$$

Ans. 2 acres 13 perches 26 yards.

2. Required the area of a triangle, having given an angle 78° 30′, and the containing sides 933 and 471 Scotch links.

Ans. 215310·89 links, = 2 acres 24 falls 17·88 ells.

3. Required the area of a triangle, two sides 12 feet 9 inches, and 7 feet 3 inches, and the included angle 57° 38′.

Ans. 5621·5 inches, = 4 yards 3 feet $5\frac{1}{2}$ inches.

4. Required the area of a triangle, an angle 54° 30′, and the containing sides 328 and 157 yards.

Ans. 20961·88 sq. yds. = 4 ac. 1 ro. 12 per. 29 yds.

5. Required the area of a triangle, an angle 128°, and the sides about it 38 and 93 poles.

Ans. 1392·416 per. = 8 ac. 2 ro. 32 per. $12\frac{1}{2}$ yds.

6. Required the area of a triangle, an angle 17° 54′, and the adjacent sides 27 and 12 miles.

Ans. 49·79184 miles.

* This rule is obvious from Prob. 3., for a triangle is half a parallelogram of the same base and altitude.

7. Required the area of a triangle, an angle 93°, and the sides about it 137 and 428 ells.

Ans. 29277·838 sq. ells = 5 ac. 13 falls 9·8 ells.

PROB. VI. Given the three sides of a triangle; to find the area.

RULE. Add the three sides together, and from half the sum subtract each side separately. Then multiply the half sum and the three remainders successively, and the square root of the last product will be the area.

That is, if a, b, c, represent the sides of the triangle, and s half their sum, then $\sqrt{\{s \times (s-a) \times (s-b) \times (s-c)\}}$ = the area.*

Or add the logarithms of the half sum and of the three remainders, and half the sum will be the logarithm of the area.

1. Required the area of the triangle ABC, of which the sides are AB 221, BC 255, and AC 238 feet.

---

* Let AB $= a$, BC $= b$, and AC $= c$, then $b : a + c :: a - c : \frac{a^2 - c^2}{b} =$ BE $-$ EC $\therefore \frac{1}{2}b + \frac{a^2 - c^2}{2b} = \frac{b^2 + a^2 - c^2}{2b} =$ BE; hence $\sqrt{\left\{a^2 - \left(\frac{b^2 + a^2 - c^2}{2b}\right)^2\right\}} = \sqrt{\frac{2a^2b^2 - a^4 + 2b^2c^2 - b^4 + 2a^2c^2 - c^4}{4b^2}} =$ AE, and by Prob. 4. the area is = AE $\times \frac{1}{2}$BC = AE $\times \frac{1}{2}b$

$$= \sqrt{\frac{2a^2b^2 - a^4 + 2b^2c^2 - b^4 + 2a^2c^2 - c^4}{16}} = \sqrt{\left(\frac{-a^2 + b^2 + c^2 + 2bc}{4} \times \frac{a^2 - b^2 - c^2 + 2bc}{4}\right)} = \sqrt{\left(\frac{a+b+c}{2} \times \frac{-a+b+c}{2} \times \frac{a-b+c}{2} \times \frac{a+b-c}{2}\right)}$$

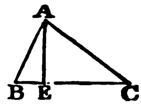
$$= \sqrt{\left\{\frac{a+b+c}{2} \times \left(\frac{a+b+c}{2} - a\right) \times \left(\frac{a+b+c}{2} - b\right) \times \left(\frac{a+b+c}{2} - c\right)\right\}}$$

$= \sqrt{\left\{s \times (s-a) \times (s-b) \times (s-c)\right\}}$ (where s is $= \frac{a+b+c}{2} =$ half the sum of the three sides of the triangle).—(See also El. Geom. 41. Cor.)

Cor. 1. The expression $\sqrt{\left(\frac{-a^2 + b^2 + c^2 + 2bc}{4} \times \frac{a^2 - b^2 - c^2 + 2bc}{4}\right)}$ becomes, by reduction, $= \frac{1}{4}\sqrt{\left\{((b+c)^2 - a^2) \times (a^2 - (b-c)^2\right\}}$, and by putting s, = the sum, and d = the difference of b and c, we obtain $\frac{1}{4}\sqrt{\left\{(s^2 - a^2) \times (a^2 - d^2)\right\}}$, which is another rule for the area.

Cor. 2. If the triangle is equilateral, and its side $= a$, the rule for the area becomes $\sqrt{(\frac{3}{2}a \times \frac{1}{2}a \times \frac{1}{2}a \times \frac{1}{2}a)} = \frac{1}{4}a^2\sqrt{3}$.

Cor. 3. If the triangle is isosceles, and each of the two equal sides be represented by a, and the other by b, the rule will be $\sqrt{\left\{\left(a + \frac{b}{2}\right) \times \frac{b}{2} \times \left(a - \frac{b}{2}\right) \times \frac{b}{2}\right\}} = \frac{b}{2}\sqrt{\left\{\left(a + \frac{b}{2}\right) \times \left(a - \frac{b}{2}\right)\right\}} = \frac{b}{2}\sqrt{\left(a^2 - \frac{b^2}{4}\right)}$.



$$(255+221+238)\times\tfrac{1}{2}=357$$
$$357-255=102$$
$$\overline{36414}$$
$$357-221=136$$
$$\overline{4952304}$$
$$357-238=119$$
$$\text{Ans. } \overline{589324176}$$

And $\sqrt{589324176}=24276$ sq. feet $=2$ ro. 9 per. $5\frac{1}{12}$ yds.

2. Required the area of a triangle, of which the sides are 834, 658, and 423 links.

The half sum	957·5	log. 2·981139
First rem.	123·5	log. 2·091667
Second rem.	299·5	log. 2·476397
Third rem.	534·5	log. 2·727948
		2)10·277151
Area 137586·3 links		log. 5·138575

$=1$ acre 1 rood 20 perches 4 yards 1·6 feet.

3. Required the area of an isosceles triangle, the equal sides 156, and the third side 78 yards.

Ans. 5890·8 yds. area, $=1$ ac. 34 per. 22 yds. 2·8 ft.

4. Required the area of an equilateral triangle, each side 34 inches. Ans. 500·56207 square inches area.

5. Required the area of a triangle, the sides 56, 52, and 60 yards. Ans. 1344 yards.

6. Required the area of a parallelogram, the sides 432 and 263, and a diagonal 342 feet.

Ans. 89945·625 sq. feet, $=2$ acres 10 perch. 11·46 yards.

7. Required the area of a triangle, one side 956 links, and each of the other two 627 links.

Ans. 1·9395567 ac. $=1$ acre 3 roods 30 perches 10 yds.

8. Required the area of a rhomboid, the sides 57 and 83 poles, and the diagonal 127 poles.

Ans. 3661·8734 per. $=22$ ac. 3 ro. 21 per. 26 yds. 3·78 ft.

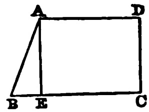
QUADRILATERALS.

Prob. VII. To find the area of a trapezoid.

Rule. Multiply half the sum of the parallel sides by the perpendicular from the one to the other.

That is, $\frac{1}{2}(AD+BC)\times AE=$ the area.

For the triangles into which it may be divided have the same *perpendicular*.



1. Required the area of the trapezoid ABCD, of which the parallel sides are AD 96 and BC 143, a third side AB 126 yards, and the perpendicular AE 89 yards.

$\frac{1}{2}(143+96)\times 89=119\cdot5\times 89=10635\cdot5$ square yards $=2$ acres 31 perches $17\frac{3}{4}$ yards.

2. Required the area of a trapezoid, the parallels 786 and 473, another side 1230, and the perpendicular distance 986 links. Ans. $6\cdot20687$ ac. $=6$ acres 33 perches 3 yards.

3. Required the area of a trapezoid, the parallels 564 and 348, a third side 452, and the perpendicular 397 feet.

Ans. 181032 sq. feet, $=4$ acres 24 perches $28\frac{5}{8}$ yards.

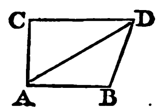
4. Required the area of a trapezoid, the parallels 93 and 157 poles, angle at the latter 62°, and the perpendicular on it 86 poles. Ans. 10750 perches, $=67$ acres 30 perches.

5. Required the area of a trapezoid, the parallel sides 386 and 294 feet, an angle at the first 43°, and the perpendicular upon the latter 328 feet.

Ans. 111520 sq. feet, $=2$ ac. 2 ro. 9 per. 18 yds. $7\frac{3}{4}$ ft.

Prob. VIII. To find the area of any quadrilateral.

Rule. Divide it into triangles, by drawing a diagonal. Find the areas of the triangles separately, and add them: the sum is the area of the figure.



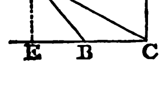
1. Required the area of the quadrilateral ABCD, of which the sides are AC 236, BD 348, AB 392, and DC 427, and the diagonal AD 473 feet.

$$\sqrt{(606\cdot5\times(606\cdot5-348)\times(606\cdot5-392)\times(606\cdot5-473)}=67003\cdot90 \text{ DAC}$$

$$\sqrt{(568\times(568-236)\times(568-427)\times(568-473)}=50259\cdot08 \text{ ABD}$$

$117262\cdot98$ square feet.

Ans. 2 acres 2 roods 30 perches 21 yards $6\frac{1}{2}$ feet.



2. Required the area of the trapeze ABCD, the sides AB 218, BC 194, CD 166 yards, and the perpendiculars from A upon BC 136, and upon CD 152 yards.

Ans. 25808 yards, $=5$ acres 1 rood 13 perches $4\frac{3}{4}$ yards.

3. Required the area of a trapeze ABCD, the sides AB 842, BC 938, CD 753, AD 826 links, and the angle A 78° 28′.

By trigonometry BD $=1055\cdot05$.

Ans. $683884\cdot54$ sq. links, $=6$ ac. 3 ro. 14 per. $6\frac{1}{4}$ yds.

4. Required the area of a trapeze ABCD, three sides AB 543, BC 428, CD 634 links, and the angles B 74° 40′ and C 84° 20′. By trigonometry BD = 729·077.

Ans. 185392·5 links, = 1 ac. 3 ro. 16 per. 19 yds.

5. Required the area of a trapeze, the four sides 328, 456, 572, and 298, and the diagonal from the angle between the first and second 598 feet.

Ans. 150274·6 sq. ft. = 3 ac. 1 ro. 31 per. 29 yds. 3·85 ft.

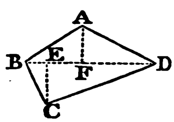
6. Required the area of a trapeze, the diagonal 1268 links, the perpendiculars from one of its extremities upon the opposite sides 784 and 672, and the length of these sides 856 and 548 links.

Ans. 519680 sq. links, = 5 ac. 31 per. 14 yds. 6·858 ft.

PROB. IX. Given a diagonal of a quadrilateral, and the perpendiculars upon it from the opposite angles; to find the area.

RULE. Add the perpendiculars together, and multiply half the sum by the diagonal.

That is, $\frac{1}{2}(AF + CE) \times BD$ = the area.*



1. Required the area of the quadrilateral ABCD, of which the sides are AB 68, BC 54, the diagonal BD 133, and the perpendiculars AF 37 and CE 44 yards.

$\frac{1}{2}(37 + 44) \times 133 = 40{\cdot}5 \times 133 = 5386{\cdot}5$ sq. yds. = 1 ac. 18 per. 2 yds.

2. Required the area of the trapeze ABCD, the sides AB 672, BC 834, the diagonal BD 1296, and the perpendiculars AE 418 and CF 550 links.

Ans. 627264 sq. links, = 6 ac. 1 ro. 3 per. 18$\frac{3}{4}$ yds.

3. Required the area of a parallelogram, of which one of the diagonals is 486 feet, and each of the perpendiculars upon it from the opposite angle 126.

Ans. 61236 sq. feet, = 1 acre 1 rood 24 perches 28 yards.

4. Required the area of a trapeze, the diagonal 1356, the angles at one of its extremities 57° and 42°, and the perpendiculars on it 586 and 724 links.

Ans. 888180 sq. links, = 8 ac. 3 ro. 21 per. 2 yds. 6 ft.

5. Required the area of a quadrilateral, of which the diagonals cut one another at right angles, the segments of the one are 328 and 523 feet, and of the other 498 and 672.

Ans. 497835 sq. ft. = 11 ac. 1 ro. 28 per. 18 yds.

* For the quadrilateral = the triangles BAD + BCD = $\frac{1}{2}$(BD × AF) + $\frac{1}{2}$(BD × CE) = $\frac{1}{2}$(AF + CE) × BD.

PROB. X. Given the diagonals of a quadrilateral, and the angle at their intersection; to find the area.

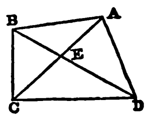
RULE I. Multiply half the product of the diagonals by the natural sine of the angle.

That is, $\frac{1}{2}(AC \times BD) \times \sin. E =$ the area.*

Or add the logarithms of one diagonal, of half the other, and the log. sine of the angle: the sum, lessened by 10 in the index, will be the logarithm of the area.

NOTE. If the angle made by the diagonals be a right angle, half the product of the diagonals is the area, for the sine of a right angle is 1.

1. Required the area of the quadrilateral ABCD, of which the diagonals are AC 674, BD 398 feet, and the acute angle at E 67° 30′.



$$
\begin{array}{r}
\text{Nat. sine of } 67^\circ\ 30' = \cdot 92388 \\
674 \\
\hline
622\cdot 69512 \\
199 \\
\hline
\end{array}
$$

Ans. Area 123916·32888 square feet, = 2 acres 3 roods 15 perches $4\frac{3}{4}$ yards.

2. Required the area of a parallelogram, the diagonals 436 and 324 yards, and their angle 48° 38′.

Ans. 53009 yards, = 10 acres 3 roods 32 perches 11 yards.

3. Required the area of a trapeze, the sides 856 and 643, the diagonal joining their extremities 1154, and the other 1345 links, and the angle made by the diagonals 57° 30′.

Ans. 654525·76 sq. links, = 6 ac. 2 ro. 7 per. $7\frac{1}{3}$ yds.

4. Required the area of a quadrilateral, the diagonals 72 and 48 feet, and containing a right angle. Ans. 192 yards.

5. The diagonals of a quadrilateral are 567 and 743 links, and they contain an angle of 73° 30′; the side joining their extremities opposite to this angle is 324. What is its area?

Ans. 201966·324 sq. links, = 2 ac. 3 per. 4 yds. $3\frac{3}{4}$ ft.

6. Required the area of a quadrilateral, the diagonals 924 links and 1256, and they bisect one another in an angle of 52° 30′.

Ans. Area 460358·7912 sq. links, = 4 ac. 2 ro. 16 per. 17 yds. 3·289 ft.

RULE II. If the sides be given instead of the diagonals.

* The triangle ACD = AED + DEC = $\frac{1}{2}$AE × ED × sin. E + $\frac{1}{2}$EC × ED × sin. E = $\frac{1}{2}$AC × ED × sin. E; and ABC = $\frac{1}{2}$AC × EB × sin. E.

Add the squares of each pair of opposite sides, and subtract the less sum from the greater: one-fourth of the remainder, multiplied by the natural tangent of the angle contained by the diagonals, will be the area.

That is, $\frac{1}{4}(AB^2 + DC^2 - BC^2 - AD^2) \times \tan. AED$ = the area.*

NOTE 1. This rule fails when the diagonals intersect at right angles, for then the tangent is infinite, and the difference of the aggregate of the squares is nothing.

NOTE 2. If a table of natural tangents be not at hand, multiply by the natural sine, and divide by the natural cos. Or add the log. of half the remainder to the log. tan.: the sum is the log. of the area.

RULE III. When the quadrilateral is in a circle, or its opposite angles are together 180°.

From half the perimeter subtract each side separately; multiply the four remainders successively, and the square root of the product will be the area. (El. Geom. 43.)

That is, if a, b, c, d be the four sides, and s half their sum, $\sqrt{\{(s-a) \times (s-b) \times (s-c) \times (s-d)\}}$ = the area.

7. Required the area of a quadrilateral, of which the sides are 7, 8, 9, and 10 yards, and the angle contained by the diagonals 80°.

$$10^2 + 8^2 = 164$$
$$9^2 + 7^2 = 130$$
$$4\,|\,34$$
$$8{\cdot}5$$
$$\text{Nat. tan. } 80° = 5{\cdot}67128$$

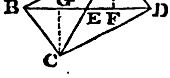
Ans. 48·20588 square yards.

8. Required the area of a trapeze in a circle, the sides 326, 438, 247, and 392 feet.

Ans. 117975·8 sq. ft. = 2 ac. 2 ro. 33 per. 10 yds. $1\frac{3}{4}$ ft.

9. Required the area of a quadrilateral in a circle, the sides 24, 26, 28, and 30 yards.

Ans. 723·98895 yards, = 23 perches $28\frac{1}{4}$ yards.



* Draw AF, CG perpendicular to the diagonal BD. Because $EF = AE \times c$ (putting c for the cosine of the angle at E), and $GE = CE \times c$; therefore $GF = AC \times c$. And because $AB^2 - AD^2 = BF^2 - FD^2$ (El. Geom. 40.) $= BG^2 + GF^2 + 2BG \times GF - FD^2$, and $DC^2 - CB^2 = DG^2 - GB^2 = DF^2 + FG^2 + 2\,DF \times FG - BG^2$ (El. Geom. 22.); therefore $AB^2 + DC^2 - AD^2 - CB^2 = 2\,FG^2 + 2\,FG \times (BG + DF) = 2FG \times (BG + GF + FD) = 2\,FG \times BD = 2\,BD \times AC \times c$; and the area $= \frac{1}{2} BD \times AC \times s$. ($s$ = sine AED); therefore $\frac{1}{4}(AB^2 + DC^2 - BC^2 - AD^2)$: the area :: c : s :: rad. : tan. AED. That is, $\frac{1}{4}(AB^2 + DC^2 - BC^2 - AD^2) \times \tan. AED$ = the area.

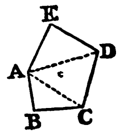
10. Required the area of a quadrilateral, of which the opposite angles are together 180°, the sides 40, 55, 60, 75 chains.
Ans. 3146·427 ch. = 314 ac. 2 ro. 22 per. 25 yds. 1·532 ft.

POLYGONS.

Prob. XI. To find the area of any rectilineal figure.

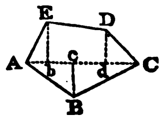
Rule. Draw diagonals so as to divide the figure into quadrilaterals and triangles, and find the areas of these figures separately, and add them: the sum is the area of the whole.

1. Required the area of the pentagon ABCDE, of which the sides are AB 354, BC 432, CD 518, DE 465, and EA 397 feet; and the diagonals AC 574, and AD 612 feet.



By Prob. VI. the triangle . .	ABC is	76338·25
	ACD	137791·11
	ADE	92302·34
Whole figure, 7 ac. 5 per. 16 yds. 6·45 ft. =		306431·70 feet.

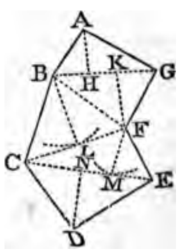
2. In order to obtain the area of the field ABCDE, I measured along the diagonal AC; and at b, 326 links from A, I took the perpendicular bE, 97 links: then I measured to c, 543 links from A, where I took the offset cB 354 links; and measuring on to d, 749 links from A, I took the offset dD 158 links. The whole diagonal AC is 987 links. Required the area.



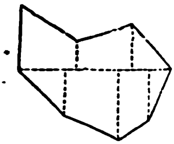
By Prob. VII. $\text{E}bd\text{D} = \frac{1}{2}(\text{E}b + \text{D}d) \times bd =$ 53932·5 links.
By Prob. IV. . . $\text{A}b\text{E} = \frac{1}{2}\text{A}b \times \text{E}b =$ 15811·0
$\text{D}d\text{C} = \frac{1}{2}dc \times d\text{D} =$ 18802·0
$\text{ABC} = \frac{1}{2}\text{AC} \times \text{B}c =$ 174699·0

Area of whole, 2 ac. 2 ro. 21 per. 5·7838 yds. = 263244·5

3. Required the area of the field ABCDEFG, of which are given the sides AB 854 and CD 927 links, the diagonals BG 1167, BF 1037, CF 1284, and CE 1342, and the perpendiculars upon BG are AH 437 and FK 384, upon CF is BL 560, and upon CE are FM 678 and DN 587 links.

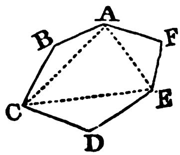


Ans. 1687388·5 links, = 16 ac. 3 ro. 19 per. 24·8534 yards.



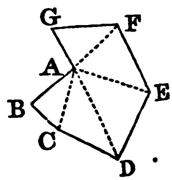
4. Measured along a diagonal from east to west, at 230 from its east extremity, a perpendicular to it on the south side, of 356 links, reached to an angle, and at 380 from the same extremity a perpendicular on the north side, of 428 reached an angle. At 673, a perpendicular of 560 reached an angle on the south side; at 812, a perpendicular of 230 reached an angle on the north; at 1140, a perpendicular of 340 reached an angle on the south; and at the west extremity 1270, there was a perpendicular of 530 on the north side.

Ans. 873572 sq. lks. = 8 ac. 2 ro. 37 per. 21 yds. 5·7132 ft.



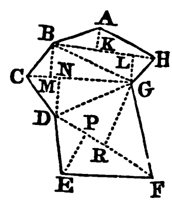
5. In a hexagon are given the sides AB 536, BC 498, CD 620, DE 580, EF 398, and AF 492 links, and the diagonals AC 918, CE 1048, and AE 652 links.

Ans. 656119·53 sq. links, = 6 ac. 2 ro. 9 per. 23 yds. 8·4173 feet.



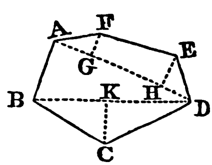
6. In a heptagon are given the sides AB 294, BC 456, CD 572, DE 640, EF 612, FG 498, and GA 386, and the diagonals AC 540, AD 864, AE 630, and AF 490 links.

Ans. 646628·38 sq. links, = 6 ac. 1 ro. 34 per. 18·3136 yds.



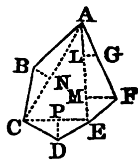
7. In an octagon, the diagonals are BH 956, BG 874, GC 1078, GD 1178, and DF 1240 links; the sides AB 620, and DE 830; and the perpendiculars AK 326, GL 520, both on BH; those on GC are BM 610, DN 354; and on DF are EP 472, and GR 396 links.

Ans. 1452144 sq. links, = 14 acres 2 roods 19 perches 13 yards.



8. Measured AB 538, and on diagonals from its extremities AG 324, and the perpendicular GF 260, AH 960, and the perpendicular HE 300; the whole diagonal AD 1240. And on the diagonal BD measured BK 460, and the perpendicular CK 350; the whole BD 1310 lks. Ans. 823855 sq. lks. = 8 ac. 38 per. 5·082 yds.

9. The diagonals are AE 810, AC 930, CE 520; on AE at 245 is perpendicular GL 65, at 440 is perpendicular FM 198, on AC at 300 is perpendicular BN 189, on EC at 400 is perpendicular DP 125 links, all exterior.



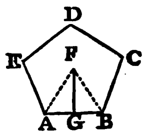
Ans. 400656·18 sq. links, = 4 acres 1 perch 1 yard 4·58 feet.

PROB. XII. To find the area of a regular polygon.

RULE. Multiply half the perimeter by the perpendicular let fall from the centre upon one of the sides.

That is, if n = the number of sides, $\frac{1}{2}n \times AB \times FG$ = the area.*

1. Required the area of the regular pentagon ABCDE, of which the side AB is 250 feet, and the perpendicular from the centre FG 172·05 feet.



Ans. $\frac{250 \times 5}{2} \times 172{\cdot}05 = 625 \times 172{\cdot}05 =$ 107531·25 square feet.

NOTE. The perpendicular may be found from the side by trigonometry; for 360° divided by twice the number of sides give the angle AFG, and its cotangent multiplied by AG gives FG the perpendicular.

2. What is the area of a regular octagon, the side 237 feet, and the perpendicular 286·084?

Ans. 271207·632 square feet.

3. What is the area of a regular hexagon, the side 356 yards, the perpendicular 308·305? Ans. 329269·74 yards.

4. What is the area of a regular heptagon, the side 237 links?

Ans. 204112·736 sq. links, = 2 ac. 6 per. 17 yds. 5 ft.

5. What is the area of a regular nonagon, the side 147 inches? Ans. 133582·32 sq. in. = 103 yds. 94·32 in.

6. What is the area of a regular decagon, the side 243 feet?

Ans. 454334·737 square feet, = 10 acres 1 rood 28 perches 24 yards 5·737 feet.

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* For the polygon may be divided, by drawing lines from the centre to its angles, into as many triangles as it has sides, all having equal bases and perpendiculars. And if s be the side of a polygon, p the perpendicular, and n the number of sides; then $\frac{1}{2}ps$ will be the area of one triangle, and $\frac{1}{2}nps$ the area of all the triangles, or of the whole polygon.

Rule II. Multiply the square of the side by the multiplier corresponding to the figure in the following Table: the product will be the area.*

Names.	No. of sides.	Angle centre.	Angle FAG.	Perpendiculars.	Multipliers.
Equilateral triangle,	3	120°	30°	0·2886751	0·4330127
Square, . . .	4	90	45	0·5000000	1·0000000
Pentagon, . . .	5	72	54	0·6881910	1·7204774
Hexagon, . . .	6	60	60	0·8660254	2·5980762
Heptagon, . . .	7	$51\frac{3}{7}$	$64\frac{2}{7}$	1·0382607	3·6339124
Octagon, . . .	8	45	$67\frac{1}{2}$	1·2071068	4·8284272
Nonagon, . . .	9	40	70	1·3737387	6·1818242
Decagon, . . .	10	36	72	1·5388418	7·6942088
Undecagon, . .	11	$32\frac{8}{11}$	$73\frac{7}{11}$	1·7028439	9·3656411
Dodecagon, . .	12	30	75	1·8660254	11·1961524

Construction of the Table. Put t for the tangent of half the angle of any regular polygon whose side is 1, and n for the number of its sides, then rad. : tan. FAG : : AG : FG; that is, $1 : t :: \frac{1}{2} : \frac{1}{2}t =$ FG, the perpendicular; hence $\frac{1}{4}tn$ = the area of the polygon: Thus the perpendicular and the area of a hexagon, whose side is 1, are $\frac{1}{2}$ tan. 60 = 0·8660254 = the perpendicular, and $\frac{1}{4}$ tan. $60 \times 6 = 0{\cdot}4330127 \times 6$ = 2·5980762 = the area.

7. Required the area of a regular heptagon, of which the side is 327 feet.

$$\begin{array}{lr} \text{Tabular multiplier} = & 3{\cdot}6339124 \\ & 327 \\ \hline & 1188{\cdot}2893548 \\ & 327 \\ \hline \end{array}$$

Ans. 388570·6190196 square feet, = 8 ac. 3 ro. 27 per. $7\frac{3}{4}$ yds.

8. What is the area of an equilateral triangle, the side 436 yards? Ans. 82313·98 yards, = 17 ac. 1 per. 3·73 yds.

9. What is the area of a regular dodecagon, the side 254 poles?

Ans. 722330·968 per. = 4514 ac. 2 ro. 10 per. 29·28 yds.

10. What is the area of a regular undecagon, the side 27 yards?

Ans. 6827·5524 sq. yds. = 1 ac. 1 ro. 25 per. 21 yds. 2·7 ft.

* Regular polygons of the same number of sides being similar, are to each other as the squares of their like sides (El. Geom. 21., Cor. 3.); now the multipliers in the Table are the areas of the polygons to the side 1, whence the rule is manifest.

11. What is the area of a regular decagon, the side 197 inches?

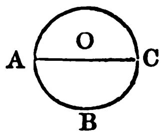
Ans. 298604·549 sq. in. = 7 per. 18 yds. 5 ft. 128·55 in.

12. What is the area of a regular nonagon, the side 254 feet? Ans. 398826·57 sq. ft. = 9 ac. 24 per. 28 yds.

OF THE CIRCLE.

Prob. XIII. Given the diameter of a circle; to find the circumference.

Rule. Multiply the diameter by $3\frac{1}{7}$, or by 3·1416; or, if greater accuracy be required, by 3·141592653, &c.*



1. Required the circumference of the circle of which the diameter is 356 yards.

	356	3·1416	3·1415926536
	$3\frac{1}{7}$	356	356
Ans.	1118·8	1118·4096	1118·4069846816

2. Required the circumference of the circle, of which the diameter is 628 links.

Ans. 1972·9248 links, = 1 furlong 38 poles 5 yds. 1·56 in.

3. Required the circumference of a circle, of which the diameter is 7958 miles.

Ans. 25000·79434 miles, = 25000 m. 6 fur. 14 pol. 1 yd.

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* It may be shown that the arc, of which t is the tangent, is $= t - \frac{1}{3}t^3 + \frac{1}{5}t^5 - \frac{1}{7}t^7$, &c. If $t = \frac{1}{2}$ the length of the arc is $\frac{1}{2} - \frac{1}{3\cdot 2^3} + \frac{1}{5\cdot 2^5} - \frac{1}{7\cdot 2^7}$, &c. $= \cdot463647609000807$, &c.; and if $t = \frac{1}{3}$, the length of the arc will be $\frac{1}{3} - \frac{1}{3\cdot 3^3} + \frac{1}{5\cdot 3^5} - \frac{1}{7\cdot 3^7}$, &c. $= \cdot321750554396641$, &c.; and the sum of these two arcs is $= \cdot785398163397448$, &c. and the tangent of their sum is $\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}$ (Trig., Formulæ XXVI.) = 1, which is the tangent of 45°. Having thus found the length of the arc of 45°, multiply it by 4, and the product 3·14159265358979З, &c. is the length of the arc of 180° when the radius is 1, or it is the circumference when the diameter is 1. And as the circumferences of circles are to one another as their radii or as their diameters (El. Geom. 45.); hence if we multiply the circumference of a circle, whose diameter is 1, by the diameter of any circle, the product will be the circumference of this circle. See Appendix.

4. Required the circumference of a circle, of which the radius is 512 feet.

Ans. 3216·9984 feet, = 4 furlongs 34 poles 5 yards 1 foot.

5. Required the circumference of a circle, of which the radius is 157 inches.

Ans. 986·4624 inches, = 4 pol. 5 yds. 1 ft. 2·46 inches.

6. Required the circumference of a circle, of which the radius is 38 poles.

Ans. 238·7616 poles, = 5 fur. 38 pol. 4 yds. 6·7968 in.

PROB. XIV. Given the circumference of a circle; to find the diameter.

RULE. Divide the circumference by 3·1416, or multiply it by ·318309886.*

1. Required the diameter of the circle, of which the circumference is 758 yards.

$$7580000 \div 31416 = 241{\cdot}2789$$
$$31831 \times 758 = 241{\cdot}2789$$

Ans. 1 furlong 3 poles 4¾ yards.

2. Required the diameter of the circle, of which the circumference is 984 links.

Ans. 313·21693 links, = 12 poles 2 yards 2¾ feet.

3. Required the diameter of the circle, of which the circumference is 24855·43 miles. Ans. 7911·72944 miles.

4. Required the diameter of the circle, of which the circumference is 398 ells Scotch. Ans. 126 ells 25·4 inches.

5. Required the diameter of the circle, of which the circumference is 928 poles.

Ans. 295·39168 poles, = 7 fur. 15 pol. 2 yds. 5·55 inches.

6. Required the diameter of the circle, of which the circumference is 1043 feet.

Ans. 331·9973 feet, = 20 poles 1·997 feet.

PROB. XV. Given the radius and the number of degrees in an arc of a circle; to find the length of the arc.

RULE. Find the circumference by Prob. XIII., multiply it by the degrees, and divide by 360°.

Or multiply the radius by the number of degrees in the arc, and by ·0174533.†

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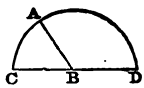
* This Prob. being the converse of Prob. 13. requires no demonstration. The number ·318309886 is the reciprocal of 3·1416.

† It has been shown that, when the radius is unity, half the circumference

1. Required the length of an arc AC of 57°, in a circle of which the radius AB is 38 feet.



$3\cdot1416 \times 38 = 119\cdot3808$ = the circumference, and $119\cdot3808 \times 57 \div 360 = 6804\cdot7056 \div 360 = 37\cdot80392$ feet.

Also $\cdot0174533 \times 57 \times 38 = 37\cdot8038478$ feet.

2. Required the length of an arc of 19° 37′, the radius being 98 yards. Ans. 33·5470317 yards.

3. Required the length of an arc of 134° 18′, the radius 9 feet 4 inches. Ans. 21·87712977 feet.

4. Required the length of an arc of 83° 24′, radius 32 poles.
Ans. 46·579367 poles = 1 fur. 6 pol. 3 yds. 6·715 in.

5. Required the length of an arc of 150°, radius 19 ells.
Ans. 49·741905 ells = 8 falls 1 ell 27·45 inches.

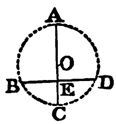
6. Required the length of an arc of 17° 50′, radius 178 miles.
Ans. 55·40259256 miles = 55 mil. 3 fur. 8 pol. $4\frac{1}{2}$ yds.

Prob. XVI. Given the chord of an arc, and its height, or the versine of its half; to find the diameter.

Rule. Divide the square of half the chord by the height, and the quotient added to the height will be the diameter.

That is, $BE^2 \div CE = AE$ (26. El. Geom., Cor. 2.)

1. Given the chord BD 287, and the height CE 78 feet; to find the diameter AC.



$287 \div 2 = 143\cdot5$, and $143\cdot5^2 \div 78 = 20592\cdot25 \div 78 = 264$, and $264 + 78 = 342$ the diameter.

2. Given the chord 178, and height 257 yards.
Ans. 287·821 yards.

3. Given the chord 843, and height 648 links.
Ans. 922·17 links, = 36 poles 4 yds. 2 ft. 7·5864 in.

4. Given the chord 40, and height 12 yards. Ans. $45\frac{1}{3}$ yds.

5. . . . . . . 560, and height 45 links. $1787\frac{8}{9}$ lks.

6. . . . . . . 325, and vers. sine 78 ells. 416·54 ells.

Prob. XVII. Given the chord of an arc, and its height; to find the length of the arc.

Rule. Find the diameter by Prob. XVI.; then, as the diameter is to the chord, so is radius to the sine of half the

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is 3·14159, &c.; hence $\frac{3\cdot14159}{180°} = \cdot01745329$, &c. is the length of an arc of 1°; therefore $r \times \cdot0174533$ = the length of 1° to radius r, and consequently if n = the number of degrees in the arc, $\cdot0174533\,rn$ = the length of that arc.

angle measured by the arc (Theor. 1. Trig.), from which find the length of the arc by Prob. XV.

1. Required the length of the arc, of which the chord is 326, and its height 97 feet.

$163^2 \div 97 = 273·90722$; and the diameter is 370·90722, and the radius 185·45361.

326 + R.	log.	12·513218
370·90722	log.	2·569265
Sin. 61° 30′ 47·2″	log.	9·943953
2		

123° 1′ 34·4″ = 123·0262°, the angle of the sector.

And 185·45361 × 123·0262 × ·0174533 = 398·2084 the arc.

2. Required the length of the arc, of which the chord is 496, and the height 654 links. Ans. 1807·787 links.

3. Required the length of the arc, of which the chord is 126, and the versed sine 14 inches. Ans. 130·10809 in.

4. Required the length of the arc, of which the chord is 78, and the versed sine 13 yards. Ans. 83·655 yards.

By Approximation. Divide the height by half the chord, and square the quotient. To 3 times this square add 15, and to the sum add 10 times the square. Then as the former sum is to the latter, so is the chord to the arc nearly.

Otherwise, having found the square as before: As $\frac{5}{9}$ of the square + 1 is to $\frac{1}{8}$ of it + 1, so is $\frac{1}{5}$ of it to a fourth number. Subtract this number from 1, multiply the remainder by the square, and to the product add 1·5: this sum, multiplied by $\frac{2}{3}$ of the chord, will produce the arc very nearly.*

5. Required the length of the arc, of which the chord is 40, and the height 6 feet.

$\frac{6}{20} = ·3$, and $·3 \times ·3 = ·09$, the square to be used: then $3 \times ·09 + 15 = 15·27 : 15·27 + ·9 = 16·17 :: 40 : 42·358$ feet the arc.

By the second approximation, $·09 \times \frac{5}{9} + 1 : ·09 \times \frac{1}{8} + 1 :: ·09 \times \frac{1}{5} : ·0173357$, and $(1 - ·0173357) \times ·09 + 1·5 = 1·58843979$, and $1·58843979 \times \frac{2}{3} \times 40 = 42·35843$.

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* Let x = the height, and $2y$ = the chord, then it may be shown that $2y \times (\frac{1}{2} + \frac{x^2}{3y^2} - \frac{x^4}{2.3y^4} + \frac{x^6}{5.7y^6} - \frac{x^8}{7.9y^8} + \&c.)$ = the length of the arc, or putting $v^2 = \frac{x^2}{y^2}$ the series becomes $2y \times (\frac{1}{2} + \frac{1}{3}v^2 - \frac{v^4}{2.3} + \frac{v^6}{5.7} - \frac{v^8}{7.9} + \&c.)$, which is very nearly equal to $2y \times \frac{15 + 13v^2}{15 + 3v^2}$, but more nearly equal to $\frac{4y}{3} \times \left(\frac{3}{2} + v^2 - v^4 \times \frac{\frac{1}{8}v^2 + 1}{\frac{5}{9}v^2 + 1}\right)$, which are the two approximations given. See Appendix.

6. Required the length of the arc of which the chord is 184, and the height 34 feet. Ans. 200·3217 feet.

7. Required the length of the arc, of which the chord is 246, and the height 534 links. Ans. 1512·00612 links.

NOTE. When the height is greater than the chord, find the diameter, and from it subtract the height, to get the height of the other segment; find its arc, and subtract it from the circumference.

8. Required the length of the arc of which the chord is 128, height 216 feet. Ans. 602·7963 feet.

9. Required the length of the arc, of which the chord is 76, height 22 links. Ans. 91·98252 links.

PROB. XVIII. Given the radius and the circumference of a circle; to find its area.

RULE. Multiply the radius by half the circumference: the product is the area.*

NOTE. The area of a semicircle is one-half, and that of a quadrant is one-fourth of the area of a circle.

1. Required the area of the circle, of which the radius is 75, and the circumference 471·24 yards.

$471{\cdot}24 \times \frac{1}{2} \times 75 = 17671{\cdot}5$ square yards, $=$ 3 acres 2 roods 24 perches $5\frac{1}{2}$ yards.

2. Required the area of the circle, of which the diameter is 10, and the circumference 31·416. Ans. 78·54.

3. Required the area of the circle, of which the diameter is 7958, and the circumference 25001 miles.

Ans. $49739489\frac{1}{2}$ miles.

4. Required the area of the circle, of which the diameter is 223, and the circumference 700 yards.

Ans. 39025 sq. yards, $=$ 8 acres 10 perches $2\frac{1}{2}$ yards.

5. Required the area of the circle, of which the diameter is 751, and the circumference 2485 feet.

Ans. $466558\frac{3}{4}$ feet, $=$ 10 ac. 2 ro. 33 per. 21 yds. $5\frac{1}{4}$ ft.

6. Required the area of the circle, of which the diameter is 169, and the circumference 532 inches.

Ans. 22477 inches, $=$ 17 yards 3 feet 13 inches.

PROB. XIX. Given the radius or diameter of a circle; to find the area.

* The circle is the limit of the polygons inscribed in it and described about it, the circumference is the limit of their perimeters, and the radius the limit of the perpendiculars; and as any polygon is $=$ perpendicular $\times \frac{1}{2}$ perimeter, therefore the circle is $=$ radius $\times \frac{1}{2}$ circumference. (El. Geom. 44.)

RULE. Multiply the square of the radius by 3·1416, or that of the diameter by ·7854.*

1. Required the area of a circle, of which the radius is 78.
Ans. $3·1416 \times 78 \times 78 = 19113·4944$.

2. Required the area of a circle, of which the diameter is 234 yards.
Ans. 43005·3624 yds. = 8 ac. 3 ro. 21 per. 20·11 yds.

3. Required the area of a circle, of which the diameter is 563 links.
Ans. 248947·4526 links, = 2 ac. 1 ro. 38 per. 9 yds. 5 ft.

4. Required the area of a circle, of which the diameter is 7·5 feet. Ans. 44·17875 feet.

5. Required the area of a circle, of which the radius is 193 yards. Ans. 117021·4584 yds. = 24 ac. 28 per. 14·46 yds.

6. Required the area of a circle, of which the diameter is 9 feet 6 inches. Ans. 70·88235 ft. = 7 yds. 7 ft. 127·06 in.

7. Required the area of a circle, of which the radius is 59 poles.
Ans. 10935·9096 per. = 68 ac. 1 ro. 15 per. 27·5129 yds.

PROB. XX. Given the circumference of a circle; to find the area.

RULE I. Divide the square of half the circumference by 3·1416.

RULE II. Multiply the square of the circumference by ·0795775 to get the area.†

1. Required the area of a circle, of which the circumference is 1284 yards.

$(1284 \div 2)^2 \div 3·1416 = 412164 \div 3·1416 = 131195·569$ yards, = 27 acres 17 perches $1\frac{1}{3}$ yards the area.

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* If R = radius, and D = diameter, then $3·1416 \times R = \frac{1}{2}$ circumference; therefore $3·1416 \times R^2 = \frac{1}{4} \times 3·1416 \times D^2 = ·7854\ D^2$, will be the area, (El. Geom. 45., Cor. 2.)

† These rules are evident from the preceding; the number ·0795775 is one-fourth of the reciprocal of 3·1416.

If D = the diameter of a circle, C the circumference, A the area, and p = 3·1416; then any two of these being given, the others may be found: Thus,

$$1.\ D = \frac{C}{p} = \frac{4A}{C} = 2\sqrt{\frac{A}{p}}.$$

$$2.\ C = pD = \frac{4A}{D} = 2\sqrt{pA}.$$

$$3.\ A = \frac{pD^2}{4} = \frac{C^2}{4p} = \frac{DC}{4}.$$

$$4.\ p = \frac{C}{D} = \frac{4A}{D^2} = \frac{C^2}{4A}.$$

2. Required the area of a circle, of which the circumference is 1386 links.

Ans. 152867·647 sq. links, = 1 ac. 2 ro. 4 per. 17·794 yds.

3. Required the area of a circle, of which the circumference is 73 feet 8 inches.

Ans. 431·8494 square feet, = 1 perch 17·733 yards.

4. Required the area of a circle, of which the circumference is 625 yards.

Ans. 31084·961 yards, = 6 acres 1 rood 27 per. 18·2 yards.

5. Required the area of a circle, of which the circumference is 1448 feet.

Ans. 166850 feet, = 3 acres 3 roods 12 per. 25 yards 8 ft.

6. Required the area of a circle, of which the circumference is 627 poles. Ans. 31284·223 per. = 195 ac. 2 ro. 4·223 p.

7. Required the area of a circle, of which the circumference is 178 inches. Ans. 2521·33 in. = 1 yd. 8 ft. 73$\frac{1}{3}$ in.

PROB. XXI. To find the area of a sector of a circle.

RULE I. If the length of the arc be known, multiply half the arc by the radius.

RULE II. If the angle of the sector be given, find the length of the arc, and work as before. Or find the area of the circle: then, as 360° to the angle of the sector, so is the area of the circle to the area of the sector.*

1. Required the area of a sector, of which the arc is 79, and the radius of the circle 47 yards.

$\frac{79}{2} \times 47 = 1856{\cdot}5$ yards, = 1 rood 21 perches 11$\frac{1}{4}$ yards.

2. Required the area of a sector, of which the arc is 17 feet 5 inches, and the radius 22 feet.

Ans. $191{\cdot}58\dot{3}$ square feet, = 21 yards $2{\cdot}58\dot{3}$ feet.

3. Required the area of a sector, of which the angle is 127° 16′, and the radius 133 feet.

The area of the circle is 55571·63245; and this, multiplied by 127$\frac{4}{15}$, and divided by 360, gives 19645·601175 sq. feet, = 1 rood 32 perches 4 yards 7·6 feet the area of the sector.

4. Required the area of a sector, of which the angle is 137° 20′, and the radius 456 links.

Ans. 249202·968 links, = 2 acres 1 ro. 38 per. 21·92 yds.

5. Required the area of a sector, of which the angle is 27°, and the radius 97 miles. Ans. 2216·94858 miles.

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* These rules are evident from those for finding the area of the circle.

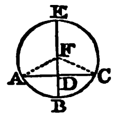
6. Required the area of a sector, of which the arc is 156 yards, the radius 478 feet.

Ans. 37284 feet, = 3 roods 16 perches 28 yards 6 feet.

PROB. XXII. To find the area of a segment.

RULE I. Find the area of the sector which has the same arc with the segment, and from it subtract the area of the triangle contained by the chord and the radii drawn to its extremities, when the segment is less than a semicircle. Otherwise, add these areas, and the remainder or the sum will be the area of the segment.

1. Required the area of the segment ABC, of which the height BD is 6, and the diameter of the circle BE 32 feet.



$\sqrt{26 \times 6} \div 16 = 12{\cdot}49 \div 16 = {\cdot}780625 =$ sin. $51{\cdot}3175°$, and $(51{\cdot}3175 \div 180) \times 3{\cdot}1416 \times 256 = 229{\cdot}289$ sector, and $229{\cdot}289 - 12{\cdot}49 \times 10 = 104{\cdot}389$ square feet the segment.

2. Required the area of the segment, of which the chord is 12, and the diameter 36 yards.

$\frac{6}{18} = {\cdot}33333$ the sine of $19{\cdot}47122°$. Ans. 8·284 yards.

3. Required the area of the segment, of which the chord is 20, and the height 2.

The diameter is 52, the angle 45·2397°. Ans. 26·87885.

4. Required the area of the segment, of which the height is 18, and the radius 56 yards.

Ans. 1024·057 sq. yards, = 33 perches 25·807 yards.

5. Required the area of the segment, of which the chord is 257, the diameter 824 feet. Ans. 3539·4216 sq. feet.

6. Required the area of the segment, of which the chord is 540, and the height 29 links.

Ans. 10464·818 links, = 16 perches 22 yards 4·475 feet.

RULE II. BY A TABLE OF SEGMENTS. Divide the height by the diameter. Look in the table for the quotient in the column of versed sines, and take out the number on the right hand of it in the column of areas, and multiply it by the square of the diameter, and the product will be the area of the segment.*

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* This rule is founded on the property, that the versed sines of similar segments are as the diameters of their respective circles, and the areas of those segments are as the squares of the diameters, which is thus proved.

NOTE. If the height be greater than the radius, subtract it from the diameter to get the height of the other segment. Find the area of this segment by the rule, and subtract it from the area of the circle to get the area of the segment required.

7. Required the area of the segment, of which the height is 18, and the diameter of the circle 48.

18 ÷ 48 = ·375, opposite to which is ·269014, and 48 × 48 × ·269014 = 619·80745 the area.

8. Required the area of the segment, of which the height is 236, and the diameter 432 links.

(432 — 236) ÷ 432 = ·4537, opposite to which is ·346465 the other segment, and ·785398 — ·346465 = ·438933 the segment required from the table. Wherefore 432^2 × ·438933 = 81915·399234 links the area, = 3 roods 11 perches 2 yds.

9. Required the area of the segment, of which the chord is 354, the height 18 feet.

Ans. 4258·128 feet, = 15 perches 19 yards 3·38 feet.

10. Required the area of the segment, of which the height is 26, and the diameter 298 yards.

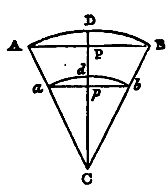
Ans. 2970·2274 yds. = 2 ro. 18 per. 5 yds. 6·546 feet.

11. Required the area of the segment, of which the radius is 125, and the height 36 links.

Ans. 4351·5625 links, = 6 perches 29·116 yards.

By Approximation. To the chord add $\frac{4}{3}$ of the chord of half the segment, and multiply the sum by $\frac{2}{5}$ of the height: the product will be the area nearly.

More accurately. Divide the height by half the chord, and square the quotient; and as 5 times the square + 11 to 4 times the square + 33, so is $\frac{1}{21}$ of the square to a fourth number. Subtract this number from 1, and multiply the remainder by the square, and to the product add 5; then multiply this sum by the chord and by the height, and $\frac{2}{15}$ of the product will be the area very nearly. See Appendix.



Let ADBA, *adba* be two similar segments cut off from the similar sectors ADBCA, *adbca* by the chords AB, *ab*, and draw the perpendicular CD to bisect them.

Then by similar triangles CA : C*a* :: CA — DP, or CP : C*a* — *dp*, or C*p* :: DP : *dp*; whence 2CA : 2C*a* : DP : *dp*.

Again, since similar sectors are as the squares of their diameters, and similar triangles as the squares of their like sides, $CA^2 : Ca^2$ :: sector CADBA :: sector C*adba* :: triangle CAB : triangle C*ab* :: segment ADBA = sector CADB — triangle CAB : segment *adba* = sector C*adb* — triangle C*ab*.

If, therefore, d be put = any diameter, and v = the versed sine, then $d : v$ :: 1 (diameter in the Table) : $v \div d$ = the versed sine of a similar segment in the Table, whose area let be called a; then $1^2 : d^2 :: a : ad^2$ = the area of the segment, whose height is v, and diameter d, which is the rule.

12. Required the area of the segment, of which the chord is 50, and the height 3.

Ans. $\sqrt{(25^2+3^2)} = 25{\cdot}1794$ the chord of $\frac{1}{2}$ the segment; then $(50+25{\cdot}1794 \times \frac{4}{3}) \times {\cdot}4 \times 3 = 100{\cdot}287$ the area nearly.

By the second method, $3 \div 25 = {\cdot}12$ and ${\cdot}12^2 = {\cdot}0144$ the square, and $5 \times {\cdot}0144 + 11 = 11{\cdot}072$: $4 \times {\cdot}0144 + 33 = 33{\cdot}0576$: : $1 \div 21 \times {\cdot}0144 = {\cdot}000\dot{6}85714\dot{2}$: ${\cdot}0020473328$ the fourth number; then $(1 - {\cdot}0020473328) \times {\cdot}0144 + 5 = 5{\cdot}014370518408$; and this, multiplied by $50 \times 3 \times 2 \div 15$, gives $100{\cdot}287410368$ the area.

13. Required the area of the segment, of which the chord is 178, and the height 14 inches. Ans. 11 ft. 85·528 in.

14. Required the area of the segment, of which the chord is 560, the height 29 poles.

Ans. 10849·8654 perches, = 67 acres 3 roods 9·8654 per.

Note. If the height be greater than half the radius, find the area of the segment subtended by the chord of half the arc, and to its double add the area of the triangle contained by the chords. To find the height of this small segment: Having found the chord of half the arc for the chord of it, multiply it by half the chord of the given segment, and subtract the product from the square of the chord of half the arc: the remainder, divided by twice the height, will give the height of the small segment.

15. Required the area of the segment, of which the chord is 156, and the height 32 inches.

Ans. 3437·4741 sq. inches, = 2 yds. 5 ft. 125·474 in.

16. Required the area of the segment, of which the chord is 68, and the height 48 yards.

Ans. 2886·377 square yards, = 2 ro. 15 per. 12·627 yds.

17. Required the area of the segment, of which the chord is 24, and the height 15 poles.

Ans. 303·529 sq. poles, = 1 ac. 3 ro. 23 per. 16 yds.

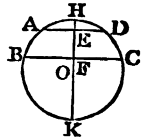
18. Required the area of the segment, of which the chord is 256, and the height 152 feet.

Ans. 32221·938 ft. = 2 roods 38 perches 10 yds. 6·44 ft.

Prob. XXIII. To find the area of a zone, or of a part of the circle intercepted between two parallels.

Rule. I. Find the areas of the segments cut off by the chords, and their difference will be the area of the zone.

Rule II. Find the area of the segment cut off by the straight line joining the extremities of the chords, and the area of the trapezoid formed by the chords; and the double of the segment added to the trapezoid will be the area of the zone.



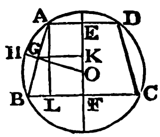
1. Required the area of the zone ABCD, of which the distance OE of the chord AD from the centre is 44, the distance OF 13, and the diameter HK 104 yards.

$(52-13)=39 \div 104 = \cdot 375$ vers. sin. to seg. $\cdot 269014$
$(52-44)= \;\; 8 \div 104 = \cdot 076923$ . . . $\cdot 027780$

Difference of segments, $\cdot 241234$
$104^2 = \;\; 10816$

Area of the zone 2 ro. 6 per. 7·679 yds. = 2609·17905 yds.

2. Required the area of a zone, of which the chords are AD 15 and BC 20, and their distance EF $17\frac{1}{2}$ feet.



Let O be the centre of the circle, join AB, and draw OG perpendicular to AB, meeting the circle in H. Draw GK parallel to AD, and AL parallel to EF; then $GK = \frac{1}{2}(AE + BF) = 8\frac{3}{4}$, and $BL = BF - AE = 2\frac{1}{2}$. Also, $AL : LB :: GK : KO = 1\frac{1}{4}$ (El. Geom. 19.), and $OF = FK - KO = 7\frac{1}{2}$. Now $OG^2 = OK^2 + KG^2$ (El. Geom. 22., Cor. 2.), therefore $OG = 8\cdot 838834765$; and $OB^2 = OF^2 + FB^2$ (El. Geom. 22., Cor. 2.), therefore OB or $OH = 12\cdot 5$, and $GH = 3\cdot 661165$, which, divided by 25, gives ·1464466 for the versed sine, for which the area is ·071350; and this, multiplied by 25^2, gives 44·59346, the area of the segment AHB, and the trapezoid $ABCD = \frac{1}{2}EF \times (AD + BC) = 306\cdot 25$, which, added to twice the segment, gives the zone 395·4369 square feet.

3. Required the area of a zone, having the parallel chords 96 and 60, and their distance 26 yards.

Ans. 2136·76 sq. yards, = 1 ro. 30 per. 19·26 yds.

4. Required the area of a zone, the parallels each 36, and their distance 84 feet.

Ans. 6380·828 sq. feet, = 23 per. 13 yds. 2·327 feet.

5. Required the area of a zone, the parallels 136 and 68, and their distance 248 feet.

Ans. 55655·1965 sq. ft. = 1 ac. 1 ro. 4 per. 12 yds. 8·2 ft.

6. Required the area of a zone, the parallels 157 and 216, and their distance 128 yards.

Ans. 27507·337944 yds. = 5 ac. 2 ro. 29 per. 10·088 yds.

7. Required the area of a zone, the parallels 247 and 192, and their distance 368 feet.

Ans. 135521·597 feet, = 3 acres 17 per. 23 yds. 6·35 feet.

8. Required the area of a zone, the parallels 32 and 40, and their distance 72 inches.

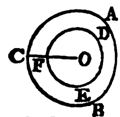
Ans. 4890·236 inches, = 33 feet $138\frac{1}{4}$ inches.

PROB. XXIV. To find the area of a ring contained by two concentric circles.

RULE I. Multiply the sum of the diameters by their difference, and then by ·7854.

RULE II. If the circumferences or similar arcs of the circles be given, multiply half their sum by the difference of the radii: the product will be the area of the ring, or of the part of it contained by the similar arcs.*

1. Required the area of the ring ABC—DEF, of which the diameters are 10 and 6, or OC 5 and OF 3.



$(10+6)(10-6)\times{\cdot}7854 = 50{\cdot}2656$ the area of the ring.

2. Required the area of the ring, of which the radii are 36 and 24 feet. Ans. 2261·952 sq. feet, = 8 per. $9\frac{1}{3}$ yds.

3. Required the area of the ring, of which the radii are 10 and 6, and similar arcs 15 and 9. Ans. 48.

4. Required the area of the ring, of which the radii are 157 and 128 yards.

Ans. 25965·324 sq. yards, = 5 ac. 1 ro. 18 per. 10 yards 7·416 feet.

5. Required the area of the ring, of which the diameters are 246 and 228 inches.

Ans. 6701·0328 inches, = 46 feet 77·0328 inches.

PROB. XXV. To find the area of a space bounded on one side by a curve-line.

RULE I. Let perpendiculars be erected upon the base, so numerous, that the part of the curve between any two nearest to one another shall differ very little from a straight line. Then add the perpendiculars at the extremities of the base, if there are any, and to half their sum add the rest of the perpendiculars. Multiply the sum by the base, and divide the product by the number of parts into which the base is divided by the perpendiculars: the quotient will be the area nearly.†

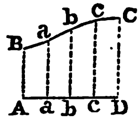
* The ring is evidently equal to the difference of the areas of the two circles; consequently let D and d be = the diameter, and $a = {\cdot}7854$, the ring will be $= aD^2 - ad^2 = a\times(D+d)\times(D-d)$, which affords the first rule.

Again, the circumferences C, c are $= 4aD$, $4ad$; whence $a\times(D+d) = \frac{1}{4}C + \frac{1}{4}c$. Substituting this in the last expression we obtain $a\times(D+d)\times(D-d) = (\frac{1}{4}C+\frac{1}{4}c)\times(D-d) = (\frac{1}{2}C+\frac{1}{2}c)\times(\frac{1}{2}D-\frac{1}{2}d)$, which is the second rule.

† The rule supposes the figure to be divided into trapezoids, and would be exact if the breadths of the trapezoids were all equal. But the common rule

RULE II. If the distances between the perpendiculars be equal, the curvature, if single, may be considered as parabolical. And, taking care to have an odd number of perpendiculars, add the first and last perpendiculars into one sum, the second, fourth, &c. into another, and all the rest into a third sum; then add the first sum, twice the third, and four times the second sum together, multiply this by the base, and divide by three times the number of parts into which the base is divided. The quotient is the area.*

NOTE. When the offset meets the base at one end, the perpendicular there must be considered $= 0$; and when it meets the base at both ends, the first and last must both be considered $= 0$; and we must always begin with the smallest perpendicular.



1. Suppose the perpendiculars at the extremities of the base to be 10 and 16, and the other perpendiculars to be 11, 14, 16, and the base to be 20 feet.

By Rule I. $(10+16) \div 2 = 13$ and $(13 + 11 + 14 + 16) \times 20 \div 4 = 270$ square feet the area.

By Rule II. $\{(10 + 16) + (11 + 16) \times 4 + 14 \times 2\} \times 20 \div 12 = 162 \times 20 \div 12 = 270$ square feet the area.

2. A curve-lined space meets the base at one of its extremities, and the perpendicular at the other extremity is 96; the other perpendiculars are 83, 70, 64, 51, 38, 25, and the base 325 links. What is the area?

Ans. $17596\frac{3}{7}$ square links; by Rule II. 16250 square links.

3. An offset meets the base at both extremities; the base is 252 links, and the perpendiculars are 24, 36, 42, 54, 67, 76, 58, 49, 33, and 19. Required the area.

Ans. $10492\frac{4}{11}$ sq. links; and by Rule II. 10416 sq. links.

4. Perpendiculars were raised from the base to a curve; those at the ends were 364 and 578, the others were 396, 418, 453, 512, 554 links, the base 1260 links.

Ans. 588840 square links; by Rule II. 588980 square links.

5. A curve meets the base at one extremity, the base is 2364, the perpendicular at the other extremity 758, and the others are 642, 587, 524, 432, 417, and 335 links.

Ans. $1119860\frac{4}{7}$ sq. links; by Rule II. $1051417\frac{1}{7}$ sq. links.

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is to add all the perpendiculars, and to multiply by the base, and divide by the number of perpendiculars; which is not much easier, and gives the answer sometimes considerably erroneous. Thus the third example would come to 11541·6.

* For the demonstration of this rule see Appendix.

MENSURATION OF SOLIDS.

DEFINITIONS.

1. A PRISM is a solid of which the ends are equal, similar, and parallel rectilineals; and the other sides are parallelograms.

NOTE. If the ends are parallelograms, the prism is called a *Parallelopiped*; and when all its sides are squares, it is called a *Cube*.

2. A CYLINDER is a round solid of uniform thickness, of which the bases are equal and parallel circles.

3. A PYRAMID is a solid which has a rectilineal figure for its base, and its sides are triangles, which have a common vertex.

4. A CONE is a round solid, which has a circle for its base, and tapers uniformly to a point at the top.

5. A SEGMENT of a solid is the part cut off from the top by a plane parallel to its base.

6. A FRUSTUM is the part left at the bottom after the segment has been cut off.

7. A WEDGE has a rectangle for its base, and its opposite side is a straight line parallel to the base, called its *Edge*.

8. A PRISMOID has any dissimilar, parallel, plane figures, of the same number of sides, for its two ends, and its upright sides trapezoids.

9. A SPHERE, or GLOBE, is a solid bounded by a curve surface, every point of which is equally distant from a point within it called the centre.

NOTE. A Sphere may be conceived to be generated by a semicircle revolving about its diameter.

10. THE AXIS or DIAMETER of a Sphere is a straight line passing through the centre, and both ends terminating at the surface.

11. A CIRCULAR SPINDLE is a solid generated by the revolution of a segment of a circle about its chord.

12. An UNGULA, or HOOF, is a part of a solid cut off by a plane inclined to the base.

13. The SOLID CONTENT of a body is the number of cubical inches, feet, &c. which the body contains.

14. A CUBICAL INCH is a solid contained by six square inches; or it is a solid, of which the length, breadth, and thickness, are each of them an inch. And the same is to be understood respecting a cubical foot, yard, &c.

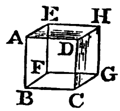
TABLE OF CUBICAL MEASURE.

1728	cubical inches	make	1	cubical foot.
27	. . feet	. .	1	yard.
$166\frac{3}{8}$	. . yards	. .	1	pole.
64000	. . poles	. .	1	furlong.
512	. . furlongs	.	1	mile.

PROB. I. To find the surface of a prism.

RULE. Find the area of one of its ends, and to its double add the sum of the areas of the parallelograms.*

1. Required the surface of a cube, upon a line of 37 inches.

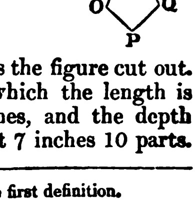


Ans. $37 \times 37 = 1369$ sq. in. area of one face, and $1369 \times 6 = 8214$ square inches whole surface.

2. Required the surface of a rectangular parallelopiped, of which the length is 11 feet, and each side of the base 27 inches. Ans. 109·125 sq. feet.

3. Required the surface of a pentagonal prism, the length 14 feet, and each side of the base 33 inches. Ans. 218·5222 ft.

TO FORM A PRISM WITH PASTEBOARD.



Let ABCD be one of the parallelograms of which the sides are compounded, AB the length, and AD a side of the base. Extend AD and BC, and make the parallelograms DK, AL, FM, &c. each equal to AC, and upon AD and BC make figures equal to the bases.

Then if the figure thus formed be cut out of the pasteboard, and folded at the sides of the parallelograms till they meet, the prism will be formed, and its surface is the figure cut out.

4. Required the surface of a chest, of which the length is 7 feet 8 inches, the breadth 4 feet 7 inches, and the depth 2 feet 9 inches. Ans. 137 feet 7 inches 10 parts.

* The truth of this rule is manifest from the first definition.

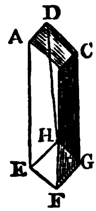
5. Required the surface of a triangular prism, of which the length is 13 feet, and the sides of the base 23, 34, and 19 inches. Ans. 85·224091 square feet.

Prob. II. To find the solid content of a prism.

Rule. Find the area of one of the ends, and multiply it by the length or perpendicular height.*

1. Required the solid content of a triangular prism, of which the height is 9 feet, and each side of the base 34 inches.

Ans. Tabular Mult. $0·4330127 \times 34^2 \times 9 \div 144 = 4505·0641308 \div 144 = 31·2851676$ cubic feet the content.



2. Required the solid content of a rectangular cistern, of which the length is 3 feet 2 inches, the breadth 2 feet 8 inches, and the depth 2 feet 6 inches. Ans. 21 feet 1 inch 4 parts.

3. Required the solid content of a heptagonal prism, of which the length is 21 feet, and each side of the base 43 inches.
Ans. 979·8693346 cubic feet.

4. Required the solid content of a pentagonal prism, the length 23 feet, and each side of the base 54 inches.
Ans. 801·312349 cubic feet.

5. Required the solid content of a quadrilateral prism, the length 19 feet, the sides of the base 43, 54, 62, and 38, and the diagonal between the first and second 70 inches.
Ans. 306·04744 cubic feet.

Prob. III. To find the surface of a cylinder.

Rule. Multiply the circumference of the base by the height: the product is the curve surface, to which add the areas of the two bases.†

1. What is the curve surface of a cylinder, of which the length is 16 feet, and the diameter of the base 27 inches?
Ans. $3·1416 \times 2\frac{1}{4} \times 16 = 113·0976$ square feet the surface.

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* If the height be one foot, it is evident that the solid will contain as many cubical feet as there are square feet in the base; if the height be two feet, the solid will contain twice as many cubical feet; if the height be three feet, it will contain three times as many, and so on.

† The truth of this rule is evident; for, if the circumference of the base be supposed to move in a direction parallel to itself, it will thus generate the convex surface of the cylinder.

2. Required the whole surface of a cylinder 13 feet long, and the circumference of its base 57 inches.

Ans. 65·3409347 square feet.

3. Required the whole surface of a cylinder, the length 12 feet, and the radius of the base 23 inches. Ans. 24133·7664 in.

4. Required the curve surface of a cylinder, the length 15 feet, and the diameter of the base 33 inches.

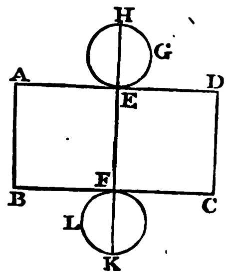
Ans. 129·591 square feet.

5. How often must a cylinder, 5 feet 3 inches long, and the diameter of its base 21 inches, revolve, to roll an acre?

Ans. 1509·18 times.

TO FORM A CYLINDER WITH PASTEBOARD.

Find the circumference of the base, and make the rectangle ABCD, of which AD is the circumference, and AB the length of the cylinder; and draw EF parallel to AB, and make EH, FK, each the diameter of the base, and describe the circles EGH and FKL. The figure thus formed being cut out of the paper, and bended round, so that AB meet CD, will form the cylinder. The area of the figure is the surface of the cylinder.



PROB. IV. To find the solid content of a cylinder.

RULE. Find the area of the base, and multiply it by the perpendicular height or length.*

1. Required the solid content of the cylinder, of which the length is 9 feet, and the circumference of the base 6 feet.

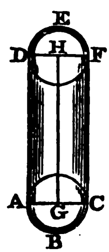
Ans. $0·795775 \times 36 \times 9 = 25·7831$ cubic feet the content.

2. Required the solid content of the cylinder, of which the length is 11 feet, and the diameter of its base 38 inches.

Ans. $·7854 \times 3\frac{1}{6} \times 3\frac{1}{6} \times 11 = 86·63398$ cubic feet.

3. Required the solid content of an oblique cylinder, the axis of which makes an angle of 75° with the base, the axis and the circumference of the base being each 20 feet.

Sin. $75° = ·965926 \times 20 = 19·31852$ the perpendicular height. Ans. 614·9278 cubic feet.



* This is proved the same way as in Prob. II.

4. An upright cylinder 20 feet high, and the diameter of the base 3 feet, is cut by a plane parallel to the axis, and 12 inches from it. Required the content of each of its segments.

Ans. 15·48738 and 125·88462 cubic feet.

5. Required the solid content of an upright cylinder 24 feet high, and the diameter of the base 27·713 inches.

Ans. 100·532253 cubic feet.

6. Required the solid content of an oblique cylinder, of which the axis inclines in an angle of 60°, the length 25 feet, and the diameter of the base 30 inches.

Ans. 106·2775055 cubic feet.

7. Required the solid content of an oblique cylinder, of which the length is 18 feet, the diameter of the base 31·305 inches, and the inclination of the axis 56°.

Ans. 79·7632337 cubic feet.

PROB. V. To find the surface of a pyramid.

RULE. Find separately the area of the base, and the areas of the triangles which constitute its sides, and add them: the sum will be the whole surface.

1. Required the surface of a triangular pyramid, of which each side of the base is 32 inches, and the perpendicular from the vertex upon a side of the base $11\frac{1}{2}$ feet.

Ans. Tabular Mult. $\cdot4330127 \times 32^2 \div 144 = 3\cdot0792$ feet area of the base; and 11 ft. 6 in. $\times$ 1 ft. 4 in. $\times 3 = 46$ feet area of the sides; then $3\cdot0792 + 46 = 49\cdot0792$ square feet whole surface.

2. What is the surface of a square pyramid, each side of the base 28 inches, and the perpendicular upon a side from the vertex 9 feet? Ans. $47\frac{4}{9}$ square feet.

3. What is the surface of a pentagonal pyramid, the slant perpendicular from the vertex 10 feet, and a side of the base 26 inches? Ans. 62·24335 square feet.

4. What is the whole surface of a triangular pyramid, of which the slant height is 18 feet, and each side of the base 42 inches? Ans. 99·80425 square feet.

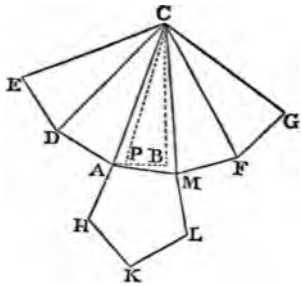
5. What is the whole surface of a hexagonal pyramid, each side of the base being 36 inches, and the slant height 20 feet?

Ans. 203·383 feet.

6. What is the whole surface of a rectangular pyramid, the sides of the base 40 and 30 inches, and the slant height upon the greater side 20·04, and upon the less side 20·07 feet?

Ans. $125\cdot308\dot{3}$ feet.

TO FORM A PYRAMID WITH PASTEBOARD.

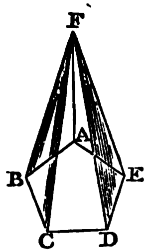


Draw AB, and BC perpendicular to it; make AB the radius of the circle circumscribing the base, and PB the radius of the inscribed circle. Then if the axis of the pyramid be given, make BC equal to it; or if the slant perpendicular be given, make PC equal to it; or if the slant side be given, make AC equal to it, and from C describe an arc through A; in this arc place AD, DE, AM, MF, &c. each equal to a side of the base, then join CD, CE, CM, &c. and upon AM make the base AHKLM. This figure being cut out, and folded along the lines till the sides meet, will form the pyramid, and its area is therefore the surface.

Prob. VI. To find the solid content of a pyramid.

Rule. Find the area of the base, and multiply it by the height, and one-third of the product will be the content. (Theorem II. Cor. 1. page 199.)

1. Required the content of a square pyramid, of which the perpendicular height is 14 feet, and a side of the base 43 inches.



	F.	I.	
	3	7	
	3	7	
	12	10	1
$14 \times \frac{1}{3} =$	4	8	
Ans. Content	59	11	

2. Required the content of a pentagonal pyramid, the height 12 feet, each side of the base 24 inches.

Ans. 27·5276384 cubic feet.

3. Required the content of a hexagonal pyramid, of which the axis is 9 feet, and each side of the base 29 inches.

Ans. $2·5980762 \times 29 \times 29 \times 9 \times \frac{1}{3} \div 144 = 45·52046$ cub. feet.

4. Required the content of an octagonal pyramid, the axis 13 feet, and each side of the base 35 inches.

Ans. 177·9923684 cubic feet.

5. Required the content of a triangular pyramid, the height 22 feet, and each side of the base 39 inches.

Ans. 33·540442 cub. ft. = 33 cubic feet 933·884 inches.

6. Required the content of a triangular pyramid, the perpendicular height 24 feet, and the sides of the base 34, 42, and 50 inches.

Ans. 39·2354 cubic feet = 39 cubic feet 406·7712 inches.

PROB. VII. To find the surface of a cone.

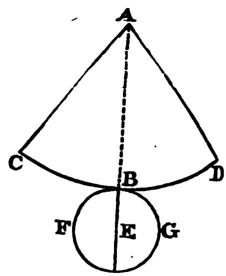
RULE. Multiply half the circumference of the base by the sum of the slant side and the radius of the base: the product is the whole surface.*

1. Required the surface of a cone, which has 10 feet for its slant side, and 32 inches for the diameter of the base.

Ans. $3.1416 \times 1\frac{1}{3} = 4·1888$ half the circumference of the base, and $(1\frac{1}{3} + 10) \times 4·1888 = 47·4731$ sq. feet surface.

TO FORM A CONE WITH PASTEBOARD.

Multiply 180° by the radius of the base, and divide it by the slant side to get the angle at the vertex. Draw AB, and make BAC and BAD each equal to the angle at the vertex. Make AB the slant side, and from A describe the arc CBD. Make BE the radius of the base, and from E describe the circle BFG. The figure thus formed is the surface of the cone; and if it be bended till AC meet AD, it will give the form of the cone.



2. Required the surface of a cone, the slant side 14 feet, and the circumference of the base 92 inches.

Ans. 58·3440553 square feet.

3. Required the surface of a cone, the slant side 10 feet, and the radius of the base 2 feet 5 inches. Ans. 94·2698 sq. ft.

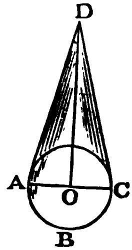
* It is evident that, if the circumference of the base be divided into an indefinite number of equal parts, and straight lines be drawn to the vertex through each point of division, the cone becomes a pyramid of an indefinite number of faces, the perpendicular height being the slant height of the cone, and the limit of the sum of the sides of its base, equal to the circumference of the base of the cone. Now, the sum of the areas of the triangles which constitute the pyramid (or the curve surface of the cone) is equal to their height, multiplied by half the sum of their bases; that is, the slant height of the cone multiplied by half the circumference of its base; but the area of the base is also equal to half the circumference multiplied by the radius; whence the whole surface is equal to half the circumference of the base multiplied by the sum of the slant side and the radius of the base.

4. Required the surface of a cone, the slant side 18 feet, and the diameter of the base 42 inches. Ans. 108·58155 sq. ft.

5. Required the surface of a cone, the slant side 9 feet, and the diameter of the base 36 inches. Ans. 49·4802 square feet.

PROB. VIII. To find the solid content of a cone.

RULE. Multiply the area of the base by the perpendicular height, and one-third of the product will be the content. (Theorem I. Cor. 2. and Theorem II. Cor. 1. p. 199.)



1. Required the content of the cone ABC–D, of which the perpendicular height DO is 14 feet, and the diameter AC of the base 43 inches.

Ans. $\cdot 7854 \times 43^2 \div 144 = 1452\cdot 2046 \div 144 = 10\cdot 084754$ sq. feet area of the base; then $10\cdot 084754 \times 14 \div 3 = 141\cdot 186556 \div 3 = 47\cdot 062185$ cubic feet content.

2. Required the content of a cone, of which the axis is 9 feet, and the circumference of the base 7 feet 10 inches.
Ans. 14·6488914 cubic feet.

3. Required the content of a cone, the slant side 15 feet, and the radius of the base 19 inches.
The axis is 178·994413 inches. Ans. 39·1591 cubic feet.

4. Required the content of a cone, the axis 18 feet, and the diameter of the base 42 inches. Ans. 57·7269 cub. feet.

5. Required the content of a cone, the diameter of the base 12·7324 feet, and the perpendicular height 107·923 feet.
Ans. 4580·40809 cubic feet.

PROB. IX. To find the surface of a frustum of a pyramid or cone.

RULE. Add the perimeters or circumferences of the two bases together, and multiply half the sum by the slant height for the upright or curve surface, to which add the areas of the two bases to get the whole surface.*

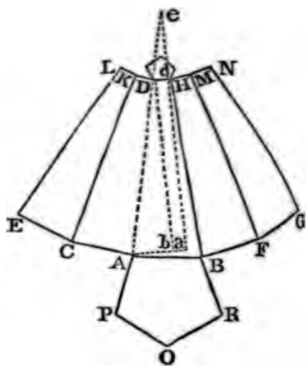
1. Required the surface of a frustum of a square pyramid, the sides of the bases being 40 and 26 inches, and the slant height 10 feet.

* This rule is evident, for the surface is composed of a number of equal trapezoids, the sums of whose parallel sides are equal to the perimeters of the ends of the frustum, and whose common height is the slant height of the frustum.

Ans. First $(40 + 26) \times 2 \times 10 = 1320$ in. surface of slant sides. Then $40 \times 40 \div 12 = 1600 \div 12 = 133\cdot\dot{3}$ inches the one base, and $26 \times 26 \div 12 = 676 \div 12 = 56\cdot\dot{3}$ inches the other; hence $(1320 + 133\cdot\dot{3} + 56\cdot\dot{3}) \div 12 = 1509\dot{6} \div 12 = 125\cdot80\dot{5}$ sq. feet whole surface.

2. Required the whole surface of a frustum of a pentagonal pyramid, the perpendicular height 11 feet, and the sides of the bases 18 and 34 inches. Ans. 137·06818 square feet.

TO FORM A FRUSTUM WITH PASTEBOARD.



Make Aa and ab equal to the radii of the circles described about the bases, and draw ad and bD perpendicular to Aa; make either bD the axis, or AD the slant side of the frustum, and produce ad and AD till they meet in e. From e describe circles through A and D, and in them place straight lines AB, AC, &c. and DH, DK, &c. equal to the sides of the bases; join BH, CK, &c.; or if the frustum be that of a cone, make aeE, aeG, the angle at the vertex. Lastly, upon AB, DH, make the bases. Then the figure will be the surface; and if it be folded along the lines, or bended, it will form the frustum.

3. Required the surface of a frustum of a cone, the diameters of the bases being 43 and 23 inches, and the slant height 9 feet. Ans. 90·72446 square feet.

4. From a cone, of which the circumference of the base is 10 feet, and its slant height 30 feet, a cone has been cut off, of which the slant side is 8 feet. Required the curve surface of the remaining frustum. Ans. $139\frac{1}{3}$ square feet.

5. Required the surface of a frustum of a cone, the perpendicular height of the frustum 13 feet, and the radii of the bases 15 and 24 inches. Ans. 150·4284 square feet.

PROB. X. To find the solid content of a frustum of a pyramid or cone.

GENERAL RULE. Find the areas of the two ends, and take the square root of their product: this added to the two

areas, and the sum multiplied by a third of the perpendicular height, will give the solid content. (Theor. II. Cor. 4. p. 199.)

That is, if A be the area of the greater end, a that of the less, and h the height, then $(A + a + \surd Aa) \times \frac{1}{3}h$ is the solidity.

PARTICULAR RULE. If the base be a circle, or a regular polygon, add a diameter, or a side of the greater base, to one of the less, and from the square of the sum subtract the product of these diameters or bases: the remainder, multiplied by the number belonging to the figure, and by a third of the height, will give the content.*

That is, using the same letters as in the demonstration in the note, $\{(A+a)^2 - Aa\} \times \frac{1}{3}ph$ is the solidity.

1. Required the content of the frustum of a square pyramid, the sides of the bases being 15 and 6 feet, and the height 24 feet.

Ans. Here $(15 + 6)^2 - (15 \times 6) = 441 - 90 = 351$ and $351 \times 1 \times 8 = 2808$ cubic feet content.

2. Required the content of the frustum of a triangular pyramid, the height of the frustum 14 feet, the sides of the greater base 21, 15, and 12, and those of the less base 14, 10, and 8 feet.

The areas of the bases are $36\surd 6$ and $16\surd 6$, and the square root of their product $24\surd 6$; therefore $(36\surd 6 + 16\surd 6 + 24\surd 6) \times \frac{1}{3} \times 14 = 868{\cdot}75236218$ cubic feet the content.

3. Required the content of the frustum of a pentagonal pyramid, the sides of the bases being 42 and 23 inches, and the height 16 feet. Ans. 207·668 cubic feet.

4. Required the content of the frustum of a cone, the diameters of the bases being 38 and 27 inches, and the height 11 feet. Ans. 63·9756 cubic feet.

5. Required the content of a mast 57 feet high, and the girths at its ends 63 and 38 inches. Ans. 81·972 cubic feet.

6. Required the content of the frustum of a cone, the height 35 feet, and the diameters of the bases 3·127 and 1·118 feet. Ans. 133·081794 cubic feet.

PROB. XI. To find the superficial and the solid contents of a wedge.

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* If A = diameter or side of the greater base, a that of the less, h the height of the frustum, and p the proper multiplier, then the height of the complete cone or pyramid is $= Ah \div d$ (putting $d = A - a$), and therefore its content is $A^2p \times Ah \div 3d = A^3ph \div 3d$.

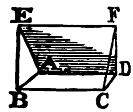
In like manner, the part of the cone which is cut off is $a^3ph \div 3d$; and therefore the content of the frustum is $(A^3 - a^3)\, ph \div 3d = \{(A + a)^2 - Aa\} \times \frac{1}{3}ph$.

RULE FOR THE SURFACE. **Find the areas of the rectangle, the two parallelograms or trapezoids, and the two triangles of which its surface consists, and add them together.**

RULE FOR THE SOLID CONTENT. **To twice the length of the base add the length of the edge, and multiply the sum by the breadth of the base, and by one-sixth of the perpendicular from the edge upon the base: the product will be the content.** (Theorem III. Cor. p. 199.)

That is, $(2BC+EF)\times AB\times\frac{1}{6}p$ is the content. ($p=$ the perpendicular.)

1. Required the superficial and the solid contents of a wedge ABCDEF, of which the sides of the base are BC 36 and BA 9 inches, the edge EF 44 inches, and the perpendicular height 22 inches.



Ans. First, $36\times 9=324=$ the rectangle, $22\times 9=198$ $=$ the two triangles, and $(36+44)\times 22=1760=$ the two trapezoids; hence $324+198+1760=2282$ square inches the whole surface.

Also (3 ft. + 3 ft. + 3 ft. 8 in.) $\times$ 9 in. $\times$ 22 in. $\div$ 6 $=$ 9 ft. 8 in. $\times$ 9 in. $\times$ 22 in. $\div$ 6 $=$ 13 ft. 3 in. 6 pts. $\div$ 6 $=$ 2 ft. 2 in. 7 pts. solid content.

2. Required the content of a wedge, of which the height is 25 inches, the edge 28 inches, and the sides of the base 34 and 10 inches. Ans. $2\cdot 314\dot{8}$ cubic feet.

3. How many solid feet are in a wedge, of which the base is 40 inches long and 10 inches broad, and each of the ends is inclined to the base in an angle of 70°, the edge being 30 inches? Ans. 1·457477 cubic feet.

4. How many solid feet are in a wedge, of which the sides of the base are 35 and 15, the length of the edge 55 inches, and the height $17\frac{3}{20}$ inches?

Ans. 5359·375 cubic inches $=$ 3 cubic feet $175\frac{3}{8}$ inches.

PROB. XII. **To find the content of any solid, of which the bases are parallel, and the greatest and least thicknesses are at its ends.**

RULE. **Find the areas of the two bases, and also the area of a section parallel to, and equidistant from, the bases; then to four times the middle area add the other two areas, and the sum, multiplied by one-sixth of the length, will give the solid content.***

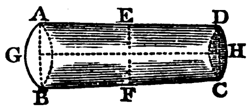
* The prismoid ABCDEFGH, see figure to example 2, may be divided into two wedges, by joining AH and BG, and if we make $EF=a$, $EH=b$, AB

That is, $(a+4b+c) \times \frac{1}{6}l =$ the content, where a and $c =$ the areas of the two bases, b the area of the middle section, and $l =$ the length of the solid.

NOTE 1. When the sides of the solid are straight between the bases, half the sum of two corresponding sides or diameters of the bases will give the corresponding side or diameter of the middle section.

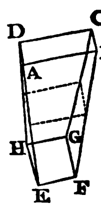
NOTE 2. When the greatest and least thicknesses are not at the ends, divide the solid into portions which shall have them at their ends. Find the contents of these portions separately, and add them: the sum will be the content of the whole.

1. A round solid ABCD, has its length GH 14 feet, the diameter of the bases AB 94, and CD 21 inches, and the diameter EF of the middle section 27 inches. Required its content.



Ans. $(94^2 + 21^2 + 54^2) \times \cdot7854 \div 6 = 12193 \times \cdot1309 = 1596\cdot0637$ and $1596\cdot0637 \times 14 \div 144 = 22344\cdot8918 \div 144 = 155\cdot17286$ cubic feet, content.

2. Required the content of the prismoid ABCDEFGH, of which the height is 22 feet, the upper base ABCD is a rectangle, of which the sides are AB 43, and BC 23 inches, and the under base EFGH a square, of which the side EF is 37 inches. Ans. $182\cdot263\dot{8}$ cubic feet.



3. Required the capacity of a cistern $47\frac{1}{4}$ inches deep, the inside dimensions are, at the top $81\frac{1}{2}$ and 55 inches, and at the bottom 41 and $29\frac{1}{2}$ inches.

Ans. $126340\cdot59375$ cubic inches, $= 455\cdot6525$ imp. gallons.

4. Required the content of a cylindroid 10 feet long, and the diameters of the bases 35 and 31 in. Ans. $59\cdot4686$ cub. ft.

5. What is the content of a log of wood, of which the length is 19 feet, and both the bases are rectangles, of which the sides of the lower are 48 and 36 inches, those of the higher 32 and 21 inches, and the sides of the middle section 45 and 34 inches? Ans. $187\cdot36\dot{1}$ cubic feet.

6. What is the content of a round solid, of which the whole length is 37 feet; the greatest girt, 77 inches, is 16 feet from the greater end, of which the girt is 54, and the middle girt

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$= m$, AD $= n$, $a + m = p$, and $b + n = q$, then p and q are double the sides of middle base. Now the under wedge is $= (m + 2a)\, b \times \frac{1}{6}h$ ($h =$ height), and the upper wedge $= (a + 2m)\, n \times \frac{1}{6}h$, whence they are together $= \{(p + a)\, b + (p + m)\, n\} \times \frac{1}{6}h = (p \times (b + n) + ab + mn) \times \frac{1}{6}h = (pq + ab + mn) \times \frac{1}{6}h$, which is the rule.

67; also, the girt at the lesser end is 36 inches, and the middle girt 59 inches. Ans. 80 cubic feet 693·5615 inches.

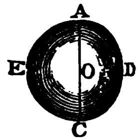
PROB. XIII. To find the surface of a sphere, or of any segment or zone of it.

RULE. Multiply the circumference of a great circle of the sphere by the axis, or by the part of it corresponding to the segment or the zone required: the product will be the surface. (Theorem IV. Cor. 2. page 201.)

NOTE. The surface of a sphere, or any part of it, cut off by a plane or planes perpendicular to the axis, is equal to the curve surface of the circumscribing cylinder, which has the same axis, or to the corresponding part of it.

1. Required the surface of a globe AECD, of which the axis AC is 18 inches.

Ans. $3·1416 \times 18^2 = 1017·8784$ square inches the surface.



2. Required the surface of a segment of a sphere, the axis 54 inches, and the height of the segment 18 inches.

Ans. 21·2058 square feet.

3. Required the surface of a zone of a sphere, the axis 72 inches, and the height of the zone 24 inches.

Ans. 5428·6848 square inches.

4. Required the surface of the moon, supposing her to be a perfect sphere, of which the diameter is 2180 miles.

Ans. 14930139·84 square miles.

5. Required the surface of the earth, supposing it to be a perfect sphere, of which the axis is 7912 miles; and also the surface of each of its zones, supposing the torrid zone to extend $23\frac{7}{15}°$ on each side of the equator, the frigid zones $23\frac{7}{15}°$ round the poles, and the breadth of each of the temperate zones to be $43\frac{1}{15}°$.

Ans. The part of the axis corresponding to each of the frigid zones is 327·192848, to each temperate zone is 2053·4668612, and to the torrid zone is 3150·67708; therefore the surface of each frigid zone is 8132797·39568, of each temperate zone is 51041592·7007, and of the torrid zone is 78314115·57481, and the whole surface is 196662895·867002 square miles.

PROB. XIV. To find the solid content of a sphere.

RULE. Multiply the cube of the axis by ·5236. (Theor. IV. page 200. A sphere is ⅔ of its circumscribing cylinder, and ·5236 is ⅔ of ·7854.)

1. Required the solidity of a sphere, of which the axis is 16 inches.

Ans. $16^3 \times \cdot 5236 = 2144 \cdot 6656$ cubic inches solidity.

2. Required the solidity of a sphere, the axis 3 feet 6 inches.

Ans. 22·44935 cubic feet.

3. Required the solidity of a sphere, the axis 19 yards.

Ans. 3591·3724 cubic yards.

4. Required the solidity of the moon, supposing her a perfect sphere, the axis 2180 miles.

Ans. 5424617475·2 cubic miles.

5. Required the solidity of the earth, supposing it to be a perfect sphere, and its axis 7912 miles.

Ans. 259332805349·80493 cubic miles.

PROB. XV. To find the solid content of a segment of a sphere.

CASE I. When the axis and the height of the segment are given.

From three times the axis subtract twice the height; multiply the remainder by the square of the height, and by ·5236: the product will be the content. (Theor. IV. Cor. 1. page 200.)

That is, if $a =$ the axis and $h =$ the height of the segment, then $(3a - 2h) \times \cdot 5236h^2$ is the solidity.

1. Required the content of a segment 13 inches high, cut off from a sphere, of which the axis is 48 inches.

$(3 \times 48 - 2 \times 13)\ 13^2 \times \cdot 5236 = 10441 \cdot 6312$ cubic inches.

2. Required the content of the frigid zone of the earth, the height 327·2, and the axis 7912 miles.

Ans. 1293874454·1815 cubic miles.

3. Required the content of a segment, of which the height is 57, and the axis 153 inches.

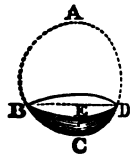
Ans. 586905·858 cub. in. = 339 cub. ft. 1113·858 inches.

4. Required the content of a segment, of which the height is $\frac{3}{8}$ of the axis. Ans. ·16567 cubes of the axis.

CASE II. When the height and the radius of the base of the segment are given.

To three times the square of the radius add the square of the height; multiply the sum by the height, and by ·5236: the product is the content. (Theor. IV. Cor. 1. page 200.)

That is, if $r =$ BE the radius of the base, and $h =$ CE the height, then $(3r^2 + h^2) \times \cdot 5236h$ is the solidity.



5. Required the content of the segment BCD, of which the height CE is 13, and the radius BE of the base 21 inches.

Ans. $(3 \times 21^2 + 13^2) \times 13 \times \cdot 5236 =$ 10155·7456 cubic inches.

6. Required the content of the segment, of which the height is 3, and the diameter of the base 9 feet. Ans. 109·5633 cubic feet.

7. Required the content of the segment, of which the height is 12, and the radius of the base 48 inches.

Ans. 44334·2592 cub. in. = 25 cub. ft. 1134·2592 inches.

8. Required the content of the segment, of which the height is 7, and the diameter of the base 84 yards.

Ans. 19575·8332 cubic yards.

PROB. XVI. To find the solid content of the middle zone of a sphere.

From the square of the axis, or greatest diameter, subtract one-third of the square of the height, then multiply the remainder by the height, and by ·7854. (Theorem IV. Cor 1. page 200.)

That is, $(a^2 - \frac{1}{3}h^2) \times \cdot 7854h$ is the solidity, where $a =$ the axis of the sphere and h the height of the zone.

NOTE. Instead of subtracting one-third of the square of the height from that of the axis, we may add two-thirds of the square of the height to the square of the least diameter.

1. Required the content of the middle zone of a sphere, of which the axis is 44, and the height of the zone 14 inches.

$(44^2 + \frac{1}{3} \times 14^2)\ 14 \times \cdot 7854 = 20569{\cdot}1024$ cubic inches.

2. Required the content of the middle zone of a sphere, of which the height is 4, and the least diameter 3 feet.

Ans. 61·7848 cubic feet.

3. Required the content of the middle zone of a sphere, of which the height is 24, and the least diameter 18 inches.

Ans. 13345·5168 cubic inches.

4. Required the content of the middle zone of a sphere, of which the height is 3, and the least diameter 5 yards.

Ans. 73·0422 cubic yards.

5. Required the solidity of the torrid zone of the earth, the axis being 7912, and the height of the zone 3150·68104 miles. Ans. 146717436810·847 cubic miles.

PROB. XVII. To find the solid content of any zone of a sphere.

Add the squares of the radii of the two ends to one-third of the square of the height; then multiply the sum by twice the height, and by ·7854. (Theorem IV. Cor. 1. page 201.)

That is, if R and r = the radii of the two ends, and h = the height, then $(R^2 + r^2 + \frac{1}{3}h^2) \times 2h \times \cdot 7854$ is the solidity.

1. Required the solid content of a spherical zone, of which the height is 10, and the diameters at its ends 12 and 8 feet.

$(6^2 + 4^2 + \frac{1}{3} \times 10^2) \times 2 \times 10 \times \cdot 7854 = 1340\cdot 416$ cub. feet.

2. Required the solid content of a spherical zone, of which the height is 14, and the diameters at its ends 16 and 12 inches. Ans. 3635·8784 cub. in. = 2 cub. ft. 179·8784 inches.

3. Required the solid content of a spherical zone, of which the height is 9, and the radii at its ends 14 and 10 yards.

Ans. 4566·3156 cubic yards.

4. Required the solid content of a spherical zone, of which the height is 11, and the diameters 18 and 13 feet.

Ans. 2826·5237 cubic feet.

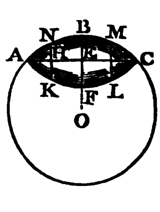
5. Required the solid content of a spherical zone, of which the height is 23, and the radii 27 and 18 inches.

Ans. 44413·8464 cub. in.

6. The height of the temperate zone of the earth is 2053·46624 miles, and the squares of the greatest and least radii are 13168239 and 2481697 square miles. Required its content. Ans. 55013866370·2 cubic miles.

PROB. XVIII. To find the solid content of a circular spindle.

RULE. Multiply the area of the generating segment by half the central distance, and subtract the product from one-third of the cube of half the length of the spindle, then four times the remainder, multiplied by 3·1416, will give the content.*



That is, if d = AE the half-length of the spindle, c = EO the central distance, a = the area ABE, and $p = 3\cdot 1416$, then $(\frac{1}{3}b^3 - ac) \times 4p$ = the solidity of the spindle ABCF.

1. Required the content of the circular spindle ABCF, of which the length AC is 40, and its greatest diameter BF 30 inches.

Ans. $20^2 \div 15 + 15 = 41\frac{2}{3}$ diam. of the circle, $20\frac{5}{6}$ the radius, $20\frac{5}{6} - 15 = 5\frac{5}{6}$ central distance, and $2\frac{11}{12}$ half the

* For the demonstration of this and the following rule, see Appendix.

central distance; then $15 \div 41\frac{2}{3} = \cdot 360$ versine of which the tabular area is $\cdot 25455$; now $\cdot 25455 \times 41\frac{2}{3}^2 \times 2\frac{1}{1}$ $= 1288\cdot 95399$, then $20^3 \div 3 = 2666\cdot\dot{6}$ and $(2666\cdot\dot{6} - 1288\cdot 95399) \times 4 \times 3\cdot 1416 = 17312\cdot 8884963$ the solid content.

2. Required the content of a circular spindle, of which the length is 24, and the greatest diameter 18. Ans. 3739·584.

3. Required the content of a circular spindle, of which the length is 32, and the greatest diameter 24 inches.

Ans. 8864·1989 cubic inches.

4. Required the content of a circular spindle, of which the length is 48, and the greatest diameter 18 inches.

Ans. 6770·97195 cubic inches.

5. Required the content of a circular spindle, of which the length is 60, and the greatest diameter 12 inches.

Ans. 3653·42525 cubic inches.

PROB. XIX. To find the solid content of the middle zone of a circular spindle.

RULE. From the square of half the length of the spindle subtract one-third of the square of half the length of the zone, and multiply the remainder by half the length of the zone; next find the area of the space which generates the zone; multiply it by the central distance, and subtract this from the former product; then twice the remainder, multiplied by 3·1416, will give the solid content.

That is, if $b =$ EH half the length of the zone, $d =$ AE half the length of the spindle, $c =$ EO the central distance, and $a =$ the generating area HNMG; then $\{(d^2 - \frac{1}{3}b^2) b - ac\} \times 2p =$ the solidity of the zone NMLK.

1. The length GH of the middle zone of the spindle ABCF is 40, and its diameters are BF 32 and KN 24 inches. Required its content.

Ans. $\frac{1}{2}(32 - 24) = 4$ and $20^2 \div 4 + 4 = 104$ diameter of circle, 52 radius and $52 - 16 = 36$ central distance; then $(104 - 16) \times 16 = 1408$ square of half the length of spindle, and $(1408 - \frac{1}{3}$ of $400) \times 20 = 25493\cdot\dot{3}$ first product. Also $4 \div 104 = \cdot 0384\frac{8}{13}$ versine of which the tabular area is $\cdot 00994$ and $\cdot 00994 \times 104^2 + (12 \times 40) = 587\cdot 51104$ generating space, which, multiplied by the central distance 36, gives $21150\cdot 39744$ second product; whence $(25493\cdot 33333 - 21150\cdot 39744) \times 2 \times 3\cdot 1416 = 4342\cdot 93589 \times 6\cdot 2832 = 27287\cdot 534784$ inches solid content.

2. Required the content of the middle zone of a circular spindle, the length 20, and the diameters 18 and 8 feet.

Ans. 3657·160776 cubic feet.

3. Required the content of the middle zone of a circular spindle, the length 36, and the diameters 24 and 16 inches.
Ans. 13090·39586778 cubic inches.

4. Required the content of the middle zone of a circular spindle, the length 60, and the diameters 50 and 30 inches.
Ans. 91302·75 cubic inches.

5. Required the content of the middle zone of a circular spindle, the length 80, and the diameters 80 and 40 inches.
Ans. 298353·77264 cubic inches.

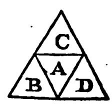
OF THE REGULAR BODIES.

A Regular Body is a solid bounded by similar and regular plane figures. Of these there can be only five.

Prob. XX. To form the five regular bodies with pasteboard.

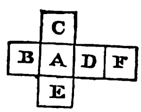
1. The Tetraedron, bounded by four equilateral triangles.

Make the equilateral triangle A, and upon each side of it make an equilateral triangle. The figure, cut out of the paper, and folded at its lines, will form the tetraedron.



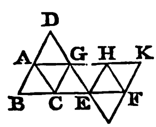
2. The Hexaedron, bounded by six squares.

Make the square A, and upon its sides the squares B, C, D, E, and on the outermost side of D make the square F. The figure, cut out and folded, will form the hexaedron.



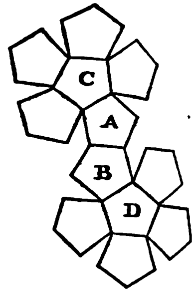
3. The Octaedron, bounded by eight equilateral triangles.

Make the equilateral triangle ABC, and through A draw AK parallel to BC, and make CE, EF, AD, AG, GH, and HK, each equal to BC, and join the points as in the figure. When folded, this figure will form the octaedron.



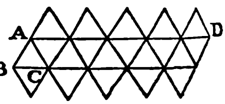
4. The DODECAEDRON, bounded by twelve pentagons.

Make two regular pentagons A and B on the same straight line, and on the most distant sides of these make the pentagons C and D; then make a pentagon on each of the sides of C and D; and the figure, when folded, will form the dodecaedron.



5. The ICOSAEDRON, bounded by twenty equilateral triangles.

Make the equilateral triangle ABC, and through A draw AD parallel to BC, and lay BC five times on each of the parallels, and join the points as in the figure. This figure, when folded, will form the icosaedron.



PROB. XXI. To find the surface and the solidity of the five regular bodies.

RULE I. TO FIND THE SURFACE. Multiply the square of the linear side by the proper number in the table under *Surface:* the product will be the surface.

RULE II. TO FIND THE SOLIDITY. Multiply the cube of the linear side by the proper number under *Solidity:* the product will be the solid content.*

TABLE

OF THE SURFACES AND SOLIDITIES OF REGULAR BODIES.

No. of faces.	Name.	Surface when the side is 1.	Solidity when the side is 1.
4	Tetraedron, . .	1·7320508	0·1178511
6	Hexaedron, . .	6·0000000	1·0000000
8	Octaedron, . .	3·4641016	0·4714045
12	Dodecaedron, .	20·6457788	7·6631189
20	Icosaedron, . . .	8·6602540	2·1816950

CONSTRUCTION OF THE TABLE. The solid content of any Regular Body is equal to its surface multiplied by $\frac{1}{3}$ of the radius of

* The truth of these rules is evident; for the surfaces of similar solids are as the squares, and their solidities as the cubes of their corresponding linear sides.

the inscribed sphere, for it is manifest that any regular solid may be divided into as many equal pyramids as it has faces, the common vertex of the pyramids being the centre of the body, which is also that of the inscribed sphere.

In the Tetraedron, which is a triangular pyramid, let e = the edge, p = the perpendicular from the vertex to the centre of the base, d = the distance from the foot of the perpendicular to one of the edges, and r = the radius of the inscribed sphere; then, since the square of the side of an equilateral triangle is equal to three times the square of the radius of the circumscribed circle, we have $e^2 = 3d^2$ and $e^2 - d^2 = p^2$, therefore $p^2 = 2d^2 = \frac{2}{3}e^2$. Or, when the edge is $= 1$, $p = \sqrt{\frac{2}{3}} = \cdot 81649658$. The area of each face is $= \frac{1}{4}\sqrt{3}$ (Cor. 2. p. 174), therefore the whole surface $= \sqrt{3} = 1\cdot 7320508$; and as $r = \frac{1}{4}\sqrt{\frac{2}{3}}$, the solidity is $\sqrt{3} \times \frac{1}{3} \times \frac{1}{4}\sqrt{\frac{2}{3}} = \frac{1}{12}\sqrt{2} = \cdot 1178511$.

The Octaedron is evidently composed of two equal square pyramids, the area of whose bases $= e^2$ and p = half the diagonal of the base $= \frac{1}{2}\sqrt{2}$, each of the faces $= \frac{1}{4}\sqrt{3}$; hence the whole surface $= 2\sqrt{3} = 3\cdot 4641016$, and the solidity $= \frac{2}{3} \times \frac{1}{2}\sqrt{2} = \frac{1}{3}\sqrt{2} = \cdot 4714045$.

The Dodecaedron is composed of 12 equal pentagonal pyramids, each of whose faces $= \frac{5}{4}\sqrt{(1 + \frac{2}{5}\sqrt{5})}$, whence the whole surface $= 15\sqrt{(1 + \frac{2}{5}\sqrt{5})} = 20\cdot 6457788$; and as $r = \sqrt{\left(\frac{25 + 11\sqrt{5}}{40}\right)} = \frac{1}{20}\sqrt{(250 + 110\sqrt{5})}$, therefore the solidity is $= 5\sqrt{\left(\frac{47 + 21\sqrt{5}}{40}\right)} = \frac{1}{4}\sqrt{(470 + 210\sqrt{5})} = 7\cdot 6631189$.

The Icosaedron is composed of 20 equal triangular pyramids, each of whose faces $= \frac{1}{4}\sqrt{3}$, hence the whole surface $= 5\sqrt{3} = 8\cdot 6602540$; and as $r = \frac{1}{2}\sqrt{\left(\frac{7 + 3\sqrt{5}}{6}\right)} = \frac{1}{12}\sqrt{(42 + 18\sqrt{5})}$, consequently the solidity $= \frac{5}{6}\sqrt{\left(\frac{7 + 3\sqrt{5}}{2}\right)} = \frac{5}{12}\sqrt{(14 + 6\sqrt{5})} = 2\cdot 1816950$.

1. Required the surface and the solidity of an octaedron, of which the side is 16 inches.

Ans. $16 \times 16 \times 3\cdot 4641016 = 886\cdot 81$ square inches surface.
$16^3 \times \cdot 4714045 = 1930\cdot 8728$ cubic inches solidity.

2. Required the surface and the solidity of a dodecaedron, of which the side is 12 feet.

Ans. Surface 2972·992 sq. ft., solidity 13241·8694592 cub. ft.

3. Required the surface and the solidity of a tetraedron, of which the side is 2 feet.

Ans. Surface 6·9282032 sq. feet, solidity 0·9428104 cub. ft.

4. Required the surface and the solidity of a hexaedron, of which the side is 27 inches.

Ans. Surface 4374 sq. inches, solidity 19683 cub. inches.

5. Required the surface and the solidity of an icosaedron, of which the side is 15 inches.

Ans. Surface 1948·55715 sq. inches, solidity 7363·220625 cubic inches.

PROB. XXII. To find the convex surface of a solid ring.

RULE. To the thickness of the ring add the inner diameter, to get the axis; multiply this by the thickness, and by $3{\cdot}1416^2 = 9{\cdot}8696$, to get the surface.*

1. Suppose the thickness of the ring 3 inches, and the inner diameter 12 inches. Required its surface.

Ans. $(12+3) \times 3 \times 9{\cdot}8696 = 444{\cdot}132$ square inches.

2. Suppose the thickness 2, and the inner diameter 18 inches. Required the surface. Ans. 394·784 square inches.

3. Suppose the thickness 3, and the inner diameter 14 inches. Required the surface. Ans. 503·3496 square inches.

4. Suppose the thickness 5, and the inner diameter 18 inches. Required the surface. Ans. 1135·004 square inches.

5. Suppose the thickness 6, and the inner diameter 24 inches. Required the surface. Ans. 1776·528 square inches.

PROB. XXIII. To find the solidity of a ring.

RULE. Multiply the axis by 3·1416 to get the length, and then multiply the length by the square of the thickness, and by ·7854: the product is the content.

Or multiply the axis by the square of the thickness, and by 2·4674.

1. Required the solidity of a ring 2 inches thick, of which the inner diameter is 18 inches.

$18+2 = 20$ axis, $20 \times 3{\cdot}1416 = 62{\cdot}832$ length.

Ans. $62{\cdot}832 \times 4 \times {\cdot}7854 = 197{\cdot}393$ cubic inches.

2. Required the solidity of a ring, the thickness 3, and the inner diameter 8 inches. Ans. 244·2726 cubic inches.

3. Required the solidity of a ring, the thickness 4, and the inner diameter 16 inches. Ans. 789·5720448 cubic inches.

4. Required the solidity of a ring, the thickness 5, and the inner diameter 12 inches. Ans. 1048·65 cubic inches.

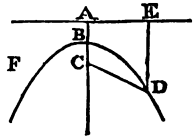
5. Required the solidity of a ring, the thickness 6, and the inner diameter 18 inches. Ans. 2131·8445 cubic inches.

* It is manifest that, as solid rings are bent cylinders, the rules for finding their surface and solidity are the same as those already given for the cylinder.

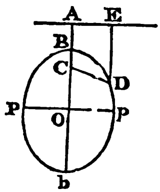
CONIC SECTIONS.

DEFINITIONS.

1. If a point D move in a plane, and its distances from a fixed point C, and from a straight line AE, both in that plane, have always the same ratio to one another, the moving point will describe a *curve*, called a *line* of the *second order*, or a *conic section*.



2. The fixed point C is called the *focus;* the straight line AE is called the *directrix;* and the constant ratio of CD to DE is called the *ratio* of the curve.

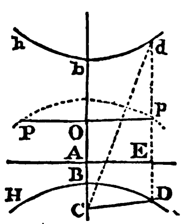


3. The straight line CA, drawn through the focus C, perpendicular to AE, is called the *axis*, or the *transverse axis*, and the point B, in which it cuts the curve, is called the *principal vertex*.

Cor. Hence CB : BA : : CD : DE, or in the ratio of the curve.

4. If CB be equal to BA, or the ratio of the curve be that of equality, the curve is called a *parabola*, as DBF.

5. If CB be less than BA, or the ratio be one of minority the curve is called an *ellipse*, as DBP.



Cor. If AC be produced beyond C to *b*, so that A*b* : *b*C : : AB : BC, the point *b* will be in the ellipse, which, therefore, contains a space.

6. If CB be greater than BA, or the ratio be one of majority, the curve is called an *hyperbola*, as DBH.

Cor. If CA be produced beyond A, so that C*b* : *b*A : : CB : BA, the point *b* will be in a hyperbola, similar, and equal to DBH, and described in the same way; it is called the *opposite hyperbola*.

7. The straight line Bb in the ellipse and hyperbola is properly the axis, B and b its vertices, and the point O in which it is bisected is called the *centre*.

8. A straight line Pp, drawn through the centre O, perpendicular to the transverse axis, is called the *conjugate* axis, and the points Pp, in the ellipse in which it meets the curve, are called its vertices. But in the hyperbola, the vertices Pp are the points in which it meets the circle described from B, with the radius OC.

9. Every straight line which is perpendicular to the directrix of a parabola, or which passes through the centre of an ellipse or a hyperbola, is called a *diameter;* and the point in which it meets the curve is its vertex.

10. A straight line which meets the curve, and does not cut it, is called a *tangent;* and if the straight line from the point of contact to the focus be parallel to the directrix, the tangent is called the *focal tangent.*

11. A straight line parallel to a tangent, is said to be *ordinately applied* to the diameter which passes through the point of contact, and the part of it between the curve and that diameter is called an *ordinate.*

12. The segments of a diameter intercepted between an ordinate and its vertices, are called *abscissae* to that ordinate.

13. Straight lines drawn through the centre of a hyperbola parallel to the straight lines which join the vertices of the axes, are called *asymptotes*.

14. Two diameters of the ellipse or hyperbola, each of which is parallel to the tangent in the vertex of the other, are called *conjugate diameters.*

15. Four times the segment of a diameter of the parabola between its vertex and the directrix, is called the *parameter* of that diameter.

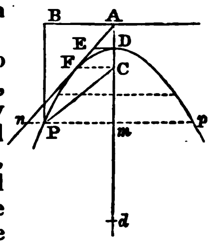
16. A third proportional to two conjugate diameters of the ellipse or hyperbola, is called the *parameter* of that diameter, which is the first of the three proportionals.

PROBLEMS.

PROB. I. To describe a conic section, of which the directrix AB, the focus C, and the ratio of the curve, are given.

Draw CA perpendicular to AB, and divide it in D, so that CD be to DA in the ratio of the curve, by (Prob. IX. Practical Geometry).

Draw BP at right angles to BA, and draw CP, so that CP : BP : : CD : DA, and let CP revolve about C, and at the same time let BP move perpendicular to AB, still retaining

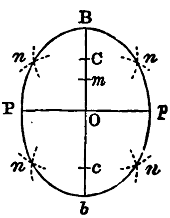


the same ratio; then their intersection P will describe the curve.

Or by points. Draw DE parallel to AB, and make it equal to DC; join AE, and produce it. Draw a great many parallels to AB, meeting AC in *m*, and AE in *n*. Take *mn* on any of them, and from the centre C cut that parallel in P and *p*; these are two points in the curve. (Con. Sec. I.) In the same manner two points may be found in every parallel, and the curve made to pass through them all.

Prob. II. Given the transverse and conjugate axes of a hyperbola or ellipse; to describe the curve.

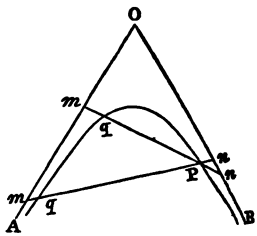
Add the squares of the two semiaxes in the hyperbola, or subtract them in the ellipse, and take the square root of the sum or remainder: this root has to the transverse semiaxis the ratio of the curve, with which the curve may be described as before; for the difference between the root and the transverse semiaxis is the distance of the focus from the principal vertex (Con. Sec. VII. formula 4.); and a fourth proportional to the root, the transverse semiaxis, and their difference, will give the distance of the directrix from the principal vertex.



Otherwise, let B*b* and P*p* be the axes, bisecting one another at right angles in the centre O. Lay BP in the hyperbola from O to C and *c*, or lay BO in the ellipse from *p* to C and *c*; then C and *c* are the foci. (Con. Sec. II.) Take any point *m* in B*b* (produced in the hyperbola), and with the distance B*m* describe two arcs *n*, *n*, from each of the foci C and *c*. With *bm* for a radius, from the foci cut these arcs in *n*, *n*, *n*, *n*; then, since in the ellipse the transverse axis is equal to the sum of two lines drawn from the foci, to meet in any point of the curve, and in the hyperbola it is equal to their difference, these will be four points of the curve. Take another point *m*, and proceed in the same manner with it to get other four points of the curve, and so on; then draw the curve through all these points.

Prob. III. Given the asymptotes and a point in the hyperbola; to describe the curve.

Let OA, OB be the asymptotes, and P the point in the curve.



Through P draw any straight line meeting the asymptotes in *m* and *n*. Make *mq* equal to *n*P, then *q* is a point in the curve. (Con. Sec. XV.) In this way any number of points in the curve may be found, and the curve drawn through them all will be the hyperbola.

MENSURATION OF CONIC SECTIONS AND THEIR SOLIDS.

DEFINITIONS.

1. A Spheroid is a solid generated by the revolution of an ellipse about one of its axes. It is called a *Prolate Spheroid* when the revolution is made about the transverse axis, and an *Oblate Spheroid* when made about the conjugate.

Note. The axis about which the ellipse revolves is called the *Axis* of the Spheroid, and the other its *Greatest Diameter.*

2. A Parabolic Conoid, or a Paraboloid, is a solid generated by a parabola about its axis.

3. A Hyperbolic Conoid, or a Hyperboloid, is a solid generated by a hyperbola about its axis.

4. Elliptic, Parabolic, and Hyperbolic Spindles, are solids formed by the revolution of these sections about a double ordinate.

Prob. I. To find the area of an ellipse.

Rule. Multiply one of the semiaxes by the other, and by 3·1416; or one of the axes by the other, and by ·7854.

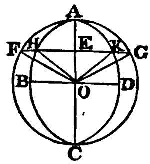
Or if the circle upon either axis be given: As that axis is to the other, so is the circle to the ellipse, and so is any sector or segment of the circle to the sector or segment of the ellipse, which has the same chord perpendicular to the first-mentioned axis.*

1. Required the area of the ellipse ABCD, of which the semiaxes are OA 436, and OB 254 feet.

Ans. $3{\cdot}1416 \times 436 \times 254 = 347913{\cdot}3504$ square feet, $= 7$ acres 3 roods 37 perches 27 yards 7 feet.

2. Required the area of an ellipse, of which the axes are 526 and 354 inches.

Ans. $146244{\cdot}6216$ sq. in. $= 112$ yards 7 feet 84·62 inches.



3. Required the area of the sector OHAK of an ellipse, the chord HK being perpendicular to the greater axis AC; the axes AC 72, BD 54, and the versed sine AE 18 feet.

The angle FOG is 120°. The circle $= 4071{\cdot}50408$, and $\frac{1}{3}$ of it $\times \frac{3}{4} = 1017{\cdot}87601536$ square feet the area of the sector.

4. Required the area of the segment of an ellipse, the chord being perpendicular to the less axis, the versed sine 12, and the axes 80 and 60 yards.

Ans. 536·7504 square yards, $= 17$ perches $22\frac{1}{2}$ yards.

5. Required the area of the segment of an ellipse, the chord being perpendicular to the greater axis, the height 25 feet, and the axes 156 and 120 feet.

Ans. 1521·936 square feet $= 5$ perches 17 yards 7·686 feet.

6. Required the area of the segment of an ellipse, the chord being perpendicular to the less axis, the height 110, and the axes 246 and 180 yards.

Ans. 22267·92492 square yards $= 4$ acres 2 roods 16 perches 3 yards 8·32338 feet.

Prob. II. To find the circumference of an ellipse.

Rule. Add the squares of the two axes, and take the square root of half the sum, and to the half of this root add a fourth of the sum of the axes, and then multiply by 3·1416: the product will be the circumference nearly.†

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* If any two straight lines be drawn perpendicular to AC, and the points be joined in which they meet the circle and the ellipse, these trapezoids are to one another as EG to EK, and their number may be multiplied, until their sum, either in the circle or ellipse, shall be more nearly equal to it than by any given difference. Therefore the circle and ellipse, which are their limits, are in that ratio; that is, the circle is to the ellipse as EG to EK, or AC : BD, or as $AC^2 \times {\cdot}7854 : AC \times BD \times {\cdot}7854$. See also Appendix.

† Let $t =$ the transverse axis, $c =$ the conjugate $d = 1 - (c^2 \div d^2)$, and $p =$ the periphery of the circumscribing circle, then it will be shown in the

That is, using the same symbols as in the demonstration $p \times \left\{\frac{t+c}{4} + \frac{1}{2}\sqrt{\left(\frac{t^2+c^2}{2}\right)}\right\}$ is the circumference.

1. Required the circumference of the ellipse, of which the axes are 24 and 18.

Ans. $\sqrt{\frac{24^2+18^2}{2}} = 21\cdot2132$, and $\frac{24+18}{2} = 21$, then $(21\cdot2132 + 21) \div 2 \times 3\cdot1416 = 66\cdot3085$ the circumference.

2. Required the circumference of the ellipse, of which the axes are 60 and 40 feet.
Ans. 158·6354 feet = 9 poles 3 yards 1 foot 1·6248 inches.

3. Required the circumference of the ellipse, of which the axes are 256 and 196 feet. Ans. 713·1156 feet.

4. Required the circumference of the ellipse, of which the axes are 320 and 240 yards. Ans. 884·1133 yards.

5. Required the circumference of the ellipse, of which the axes are 16·6 and 12·8 inches. Ans. 46·3736 inches.

6. Required the circumference of the ellipse, of which the axes are 27 and 18 poles. Ans. 71·385917 poles.

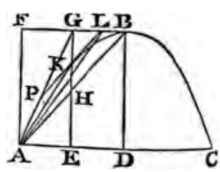
PROB. III. To find the area of a parabola.

RULE. Multiply the base by the perpendicular height, and $\frac{2}{3}$ of the product will be the area.*

Appendix that the circumference of the ellipse is $= p \times \left(1 - \frac{d}{2.2} - \frac{3d^2}{2.2.4.4} - \frac{3.3.5.d^3}{2.2.4.4.6.6} - \text{\&c.}\right)$.

But $\sqrt{(1 - \frac{1}{2}d)} = 1 - \frac{d}{2.2} - \frac{d^2}{2^3.4} - \frac{3d^3}{2^4.4.6} -$ &c. is a series which differs from the former only by the small series $-\frac{d^2}{64} - \frac{3d^3}{256} -$ &c.; rejecting this difference, therefore, we have $p \times \sqrt{(1 - \frac{1}{2}d)} =$ the circumference of the ellipse. Now as the one series gives the circumference nearly as much too large as the other gives it too small, their arithmetical mean, or $\frac{1}{2}p \times \left\{\frac{t+c}{2} + \sqrt{\left(\frac{t^2+c^2}{2}\right)}\right\}$, which is the rule, gives the circumference very accurately.

* If EG bisect AD, the triangle AFG = $\frac{1}{2}$ AFB, or it is $>$ $\frac{1}{2}$ trilineal AFBK. Also, since GK = KH, the triangle PLG = $\frac{1}{2}$ ALG, or $>$ $\frac{1}{2}$ trilineal AGBK; and every triangle thus formed cuts off more than the half of what was left by the preceding; therefore the trilineal AFBK is the limit of the sum of the triangles. Now the triangle AFG = $\frac{1}{4}$FD, and the triangle GPL = $\frac{1}{4}$AGB, or of AFG, and so on; therefore



1. Required the area of the parabola ABC, of which the base AC is 54, and the height BD 36 feet.

Ans. $\frac{2}{3} \times 54 \times 36 = 1296$ square feet area.

2. Required the area of the parabola, of which the base is 42, and the height 63 yards. Ans. 1764 square yards.

3. Required the area of the parabola, of which the base is 482, and the height 320 feet. Ans. $102826\frac{2}{3}$ square feet.

4. Required the area of the parabola, the base 126, and the height 210 inches. Ans. 17640 sq. in. = 13 yds. 5 ft. 72 in.

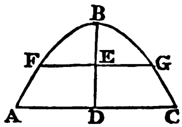
5. Required the area of the parabola, the base 67, and the height 98 yards. Ans. $4377\frac{1}{3}$ square yards.

6. Required the area of the parabola, the base 16, and the height 12 poles. Ans. 128 perches.

PROB. IV. To find the area of a frustum of a parabola.

RULE. Find a third proportional to the sum of the bases, and one of them, to which add the other base: the sum, multiplied by two-thirds of the height, gives the area.

That is, $\left(A + \frac{a^2}{A+a}\right) \times \frac{2}{3}b$, or $\left(a + \frac{A^2}{A+a}\right) \times \frac{2}{3}b =$ the area; A, a being the two ends, and b the height.*



1. Required the area of the frustum of a parabola, of which the bases are 64 and 32, and the height 26 feet.

Ans. $64 + 32 : 32 :: 32 : 10\frac{2}{3}$, and $(10\frac{2}{3} + 64) \times 26 \times \frac{2}{3} = 74\frac{2}{3} \times 17\frac{1}{3} = 1294\frac{2}{3}$ sq. feet = 4 per. 22·8 yds. the area.

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the sum of them is FD $\times \left(\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3}, \text{&c.}\right)$, and the limit of this geometrical series is FD $\times \frac{1}{4-1} = \frac{1}{3}$FD $= \frac{1}{3}$BD $\times$ AD, and therefore AKBD $= \frac{2}{3}$FD. See also Appendix.

* Let A = AC, a = FG, and b = ED, then by the property of the parabola $A^2 - a^2 : b :: A^2 : \frac{bA^2}{A^2 - a^2} :: a^2 : \frac{ba^2}{A^2 - a^2} =$ the altitudes DB and ED of the two complete segments whose bases are the ends A, a of the frustum; hence the difference of the areas of these segments = the area of the frustum AFGC. That is, $\frac{2}{3}b \times \left(\frac{A^3}{A^2 - a^2}\right) - \frac{2}{3}b \times \left(\frac{a^3}{A^2 - a^2}\right) = \frac{2}{3}b \times \frac{A^3 - a^3}{A^2 - a^2} = \frac{2}{3}b \times \frac{A^2 + Aa + a^2}{A + a} = \frac{2}{3}b \times \left(A + \frac{a^2}{A + a}\right) = \frac{2}{3}b \times \left(a + \frac{A^2}{A + a}\right)$ which affords the rule.

2. Required the area of the frustum of a parabola, of which the bases are 16 and 54, and the height 46 yards.

Ans. 1768·15238 sq. yds. = 1 ro. 18 per. 13·65 yds.

3. Required the area of the frustum of a parabola, of which the bases are 364 and 186, and the height 280 feet.

Ans. 79688·33$\frac{3}{3}\frac{1}{3}$ square feet.

4. Required the area of the frustum of a parabola, of which the bases are 424 and 268, and the height 318 inches.

Ans. 111891·8828 square inches.

5. Required the area of the frustum of a parabola, of which the bases are 63 and 22, and the height 44 poles.

Ans. 2015·024 perches.

6. Required the area of the frustum of a parabola, of which the bases are 18 and 12, and the height 20 yards.

Ans. 304 square yards.

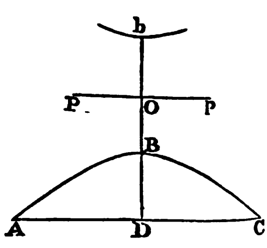
PROB. V. To find the area of a hyperbola.

RULE. Multiply half the base by the semitransverse axis, and its distance from the centre by the semiconjugate, and divide the sum of the products by the product of the two semiaxes, and take the hyperbolic logarithm of the quotient, and multiply it by the product of the semiaxes, and subtract the product from the product of half the base by its distance from the centre: the remainder will be the area.

That is, if a = BO the semitransverse axis, b = PO the semiconjugate, c = AD half the base, and d = DO the distance from the centre; then $cd - ab \times$ hyp. log. $\frac{ca + db}{ab}$ is the area.*

NOTE. The hyperbolic logarithm is got by multiplying the common logarithm by 2·30258509.

1. Required the area of the hyperbola ABC, of which the base AC is 24, the altitude BD 10, the transverse axis Bb 30, and the conjugate Pp 18 feet.



Ans. $\frac{12 \times 15 + 25 \times 9}{15 \times 9} = 3$, of which the logarithm 0·477121 $\times$ 2·30258509 = 1·0986117 the hyperbolic logarithm of 3; and

* For the demonstration of this rule see Appendix.

this logarithm, multiplied by 15×9, gives 148·3125795, which, taken from 25×12, leaves 151·6874205 sq. feet the area.

2. Required the area of the hyperbola, of which the base is 208, the height 70, and the transverse semiaxis 105 yards.

$\surd\{(210+70)\times 70)\} : 104 :: 105 : 78$ the semiconjugate.

Ans. 9202·36772 sq. yds. = 1 ac. 3 ro. 24 per. 6·3677 yds.

3. Required the area of the hyperbola, of which the base is 384, the height 250, and the axis 176 feet.

Ans. 55686·0453 square feet.

4. Required the area of the hyperbola, of which the base is 156, height 196, and axis 248 yards. Ans. 18449·697 sq. yds.

5. Required the area of the hyperbola, of which the base is 48, height 22, and axis 36 inches. Ans. 647·2532 sq. in.

6. Required the area of the hyperbola, of which the base is 96, height 110, and axis 124 poles. Ans. 6324·6852 perches.

SOLIDS.

PROB. VI. To find the solid content of a spheroid.

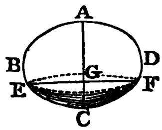
RULE. Multiply the square of the greatest diameter by the axis, and by ·5236 (or $\frac{1}{6}$ of 3·1416), the product is the content. (Theorem I. Cor. 2. page 238.)*

That is, if t = the transverse, and c = the conjugate axis of the generating ellipse; then $\cdot5236 \times tc^2$ = the oblate, and $\cdot5236 \times t^2c$ = the solidity of the oblong spheroid.

1. Required the solid content of an oblong spheroid, the axes of the generating ellipse being 54 and 36 inches.

Ans. $36^2 \times 54 \times \cdot5236 = 36643\cdot6224$ cubic in. the content.

2. Required the content of the oblate spheroid ABCD, the axes of the generating ellipse being 42 and 30 feet.



Ans. 27708·912 cubic feet.

3. Required the content of an oblong, and also of an oblate spheroid, the axes of each ellipse being 48 and 36 inches.

Ans. The oblate 43429·4784, and the oblong 32572·1088 cubic inches.

4. Required the content of an oblong spheroid, of which the axes are 50 and 30 yards. Ans. 23562 cubic yards.

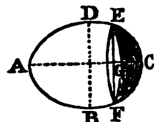
5. Required the content of an oblong, and also of an oblate spheroid, the axes of each ellipse being 25 and 15 inches.

Ans. Oblong 2954·25 cubic in. oblate 4908·75 cubic in.

* If a circle be described upon either axis of an ellipse, and both revolve about that axis, the spheroid generated by the ellipse will be to the sphere described by the circle, as the circle described by the revolving axis of the ellipse to the circle described by the diameter of the circle; and so is any segment or frustum of the spheroid to the corresponding segment or frustum of the sphere.

PROB. VII. To find the solid content of a segment of a spheroid.

RULE. Find the spherical segment which has the same height and the same axis; then, if the base be perpendicular to the fixed axis, the square of that axis is to the square of the other as the spherical to the spheroidal segment. But if the revolving axis be perpendicular to the base, that axis is to the fixed one as the spherical to the spheroidal segment. (Theorem I. Cor. 2. page 238.)



1. The height CG of the segment ECF of the oblong spheroid ABCD, of which the base is perpendicular to the fixed axis, is 16, the axes are AC 48 and BD 38 feet. Required the content.

Ans. $\{(48 \times 3) - (16 \times 2)\} \times 16^2 \times {\cdot}5236 = 112 \times 256 \times {\cdot}5236 = 15012{\cdot}6592$; then $48^2 : 38^2 :: 15012{\cdot}6592 : 9408{\cdot}97564$ cubic feet the content.

2. Required the content of a segment of an oblate spheroid, the base perpendicular to the fixed axis, the height 12, and the axes 44 and 30 inches. Ans. 10704·562176 cubic inches.

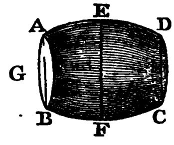
3. Required the content of a segment of an oblong spheroid, the base parallel to the fixed axis, the height 14, and the axes 54 and 45 inches. Ans. 13177·12704 cubic inches.

4. Required the content of a segment of an oblate spheroid, the base parallel to the fixed axis, the height 18, and the axes 50 and 42 feet. Ans. 16245·339264 cubic feet.

PROB. VIII. To find the solid content of the middle zone of a spheroid.

RULE. To twice the area of the greater base add the area of the less, and multiply the sum by one-third of the length or height: the product will be the solid content. (Theorem I. Cor. 1. p. 238, and Theorem IV. Cor. 1. p. 200.)

That is, if $D =$ the diameter of the greater end, and $d =$ that of the less, $a =$ the altitude, and $n = {\cdot}7854$; then $(2D^2 + d^2) \times \frac{1}{3}an =$ the solidity of the zone.



1. Required the content of the middle zone ABCD of an oblong spheroid, the bases being perpendicular to the fixed axis, the height GH 48, the greater diameter EF 42, and the less AB 32 inches.

Ans. $(42^2 \times 2 + 32^2) \times 16 \times {\cdot}7854 = 4552 \times 16 \times {\cdot}7854 = 57202{\cdot}2528$ cubic inches the content.

2. Required the content of the middle zone of an oblong

spheroid, the bases parallel to the fixed axis, the height 28, the diameters of the greater base 54 and 42, and those of the less 35 and 25 inches. Ans. 39664·7944 cubic inches.

3. Required the content of the middle zone of an oblate spheroid, the bases perpendicular to the fixed axis, the height 19, the diameter of the greater base 46, and of the less 38 feet. Ans. 28233·5592 cubic feet.

4. Required the content of the middle zone of an oblate spheroid, the bases parallel to the fixed axis, the height 12, the diameters of the greater base 35 and 50, and those of the less base 20 and 28 feet. Ans. 12754·896 cubic feet.

5. Required the content of the middle zone of an oblong spheroid, the bases perpendicular to the fixed axis, the length 40, and the diameters 30 and 18 inches.

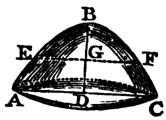
Ans. 22242·528 cu. in. = 12 cubic feet 1506·528 inches.

6. Required the content of the middle zone of an oblate spheroid, the bases parallel to the fixed axis, the length 40 inches, the diameters of the greater base 50 and 30, and of the less 30 and 18 inches. Ans. 37070·88 cubic inches.

Prob. IX. To find the solid content of a parabolic conoid.

Rule. Multiply the area of the base by half the height: the product will be the content. (Theorem II. p. 238.)

1. Required the content of the parabolic conoid ABC, of which the height BD is 36, and the diameter AC of the base 42 inches.



Ans. $42^2 \times 18 \times \cdot 7854 = 24938 \cdot 0208$ cubic in. the content.

2. Required the content of a parabolic conoid, of which the height is 54, and the diameter of the base 40 feet. Ans. 33929·28 cubic feet.

3. Required the content of a parabolic conoid, of which the height is 16, and the diameter of the base 36 inches. Ans. 8143·0272 cubic inches.

4. Required the content of a parabolic conoid, of which the height is 30, and the diameter of the base 40 inches. Ans. 18849·6 cubic inches.

5. Required the content of a parabolic conoid, of which the height is 27, and its parameter 12 inches. Ans. 13741·3584 cubic inches.

Prob. X. To find the solid content of a frustum of a paraboloid.

Rule. Multiply the sum of the squares of the diameters of the bases by half the height, and by ·7854 : the product will be the content. (Theorem II. Cor. page 238.)

1. Required the content of the frustum EACF (see last figure) of a paraboloid, of which the height DG is 12, and the radii of the bases EG 20, and AD 28 inches.

Ans. $(28^2+20^2)\times 6\times 3{\cdot}1416 = 1184\times 6\times 3{\cdot}1416 =$ 22317·9264 cubic inches the content.

2. Required the content of the frustum of a paraboloid, of which the height is 38, and the diameters of the bases 32 and 20 feet. Ans. 21249·7824 cubic feet.

3. Required the content of a cask consisting of two frustums of a parabolic conoid joined at their greatest ends, the greatest diameter 34 inches, the least 27, and the whole length 42 inches.

Ans. 31090·059 cubic inches, = 112 imperial gallons 1 pint.

4. Required the content of a cask, the length 40, and the diameters 32 and 26 inches. Ans. 26703·6 cubic inches.

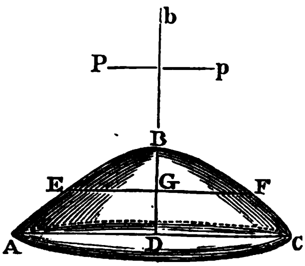
5. Required the content of a cask, the length 45, and the diameters 40 and 20 inches. Ans. 35343 cubic inches.

Prob. XI. To find the solid content of a hyperbolic conoid.

Rule. Find the content of a cylinder having the same base and altitude with the hyperboloid; then, as the sum of the transverse axis and the height is to the sum of this axis and two-thirds of the height, so is half the cylinder to the content of the hyperboloid. (Theorem III. Cor. 1. p. 239.)

1. Suppose the height BD to be 10, the radius of the base AD 12, and the transverse axis Bb 30 inches. Required the content.

$$\begin{array}{r} 3{\cdot}1416 \\ 144 \\ \hline 452{\cdot}3904 \\ 5 \\ \hline \end{array}$$



40 : 2261·952 : : $36\frac{2}{3}$: 2073·456 cubic inches.

2. Suppose the height 14, the radius of the base 48, and the transverse axis 60 feet. Required the content.

Ans. 47472·4629 cubic feet.

3. Suppose the height 22, the radius of the base 60, and the transverse axis 96 feet. Required the content.

Ans. 116675·829 cubic feet.

4. Suppose the height 49, the radius of the base 78, and the transverse axis 124 inches. Required the content.

Ans. 424069·1484 cubic inches.

5. Suppose the height 55, the radius of the base 96, and the transverse axis 84 inches. Required the content.

Ans. 691191·778 cubic inches.

PROB. XII. To find the content of a frustum of a hyperboloid.

RULE. Find a fourth proportional to the transverse, the conjugate, and the altitude, and subtract a third of its square from the sum of the squares of the radii of the bases: the remainder, multiplied by twice the altitude, and by ·7854, will give the content. (Theorem III. Cor. 2. p. 239.)

1. Suppose the transverse Bb 270, the conjugate Pp 108, the height DG 10, and the radii of the bases AD 24 and EG 16 inches. Required the content of the frustum.

Ans. 270 : 108 : : 10 : 4, and $4^2 \div 3 = 5\frac{1}{3}$; then $(24^2 + 16^2 - 5\frac{1}{3}) \times 20 \times \cdot 7854 = 826\frac{2}{3} \times 20 \times \cdot 7854 = 12985\cdot 28$ cubic inches the content.

2. Suppose the transverse 200, conjugate 350, height 14, and the radii of the bases 36 and 20 feet. Required the content.

Ans. 32897·0026 cubic feet.

3. Suppose the transverse 270, conjugate $\frac{108}{\sqrt{10}}$, height 40, diameters of the bases 32 and 24 inches. Required the content.

Ans. 24596·6336 cubic inches.

4. Suppose the transverse 30, conjugate 18, height 5, and the squares of the radii 144 and 194·4 inches. Required the content.

Ans. 2634·2316 cubic inches.

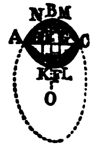
5. Suppose the transverse 45, conjugate 27, height 9, diameters 72 and 544 inches. Required the content.

Ans. 1064111·002416 cubic inches.

PROB. XIII. To find the solid content of an elliptical spindle.

RULE. Divide three times the area of the generating segment by the length of the spindle, and from the quotient subtract the greatest diameter; multiply the remainder by four times the central distance, and subtract the product from the square of the greatest diameter: the remainder, multiplied by the length and by ·5236, will give the content.*

* For the demonstration of this and the three following rules see Appendix.



1. Suppose the length AC of the spindle to be 40, the greatest diameter BF 12, the central distance OE 9 inches, and the area of the elliptic segment ABC 167·7345 square inches. Required the content.

Ans. $167{\cdot}7345 \times 3 \div 40 - 12 = {\cdot}5801$, then $12^2 - ({\cdot}5801 \times 4 \times 9) = 144 - 20{\cdot}88316 = 123{\cdot}1169$, and $123{\cdot}1169 \times 40 \times {\cdot}5236 = 2578{\cdot}55931$ cubic inches the content.

2. Let the length of the spindle be 48, its greatest diameter 18, and the central distance 24 inches. Required the content.
The elliptic segment is 296·89885. Ans. 6801·10457 cu. in.

3. Required the content of an elliptical spindle, the length 60, the greatest diameter 24, and the central distance 32 inches. Ans. 15113·986 cubic inches.

4. Required the content of an elliptical spindle, the length 36, the greatest diameter 16, and the central distance 20 inches. Ans. 4039·5446784 cubic inches.

5. Required the content of an elliptical spindle, the length 30, the greatest diameter 14, and the central distance 20 inches. Ans. 2565·4321308 cubic inches.

PROB. XIV. To find the content of the middle zone of an elliptical spindle.

RULE. Find the area of the elliptical segment, of which the chord is equal to the length of the zone, divide three times this area by its length, and from the quotient subtract the difference between the greatest and least diameters of the zone, and multiply the remainder by eight times the central distance. Subtract the product from the sum of twice the square of the greatest diameter and the square of the least; the remainder, multiplied by the length and by ·2618, will give the content.

NOTE. The rules for an elliptical spindle and its zones will give the content of a hyperbolical spindle and of its zones, if the product be added to the squares of the diameters instead of subtracting it.

1. Suppose the length GH of the zone (see last figure) to be 40, its greatest and least diameters FB 32, and KN 24, the central distance OE 4 inches, and the area of the elliptical segment cut off by the straight line KL 109 square inches. Required the content of the zone.
Ans. $(109 \times 3 \div 40 - 8) \times 4 \times 8 = 5{\cdot}60$; then $\{(40^2 \times 2 + 24^2) - 5{\cdot}6\} \times 40 \times {\cdot}2618 = (2624 - 5{\cdot}6) \times 10{\cdot}472 = 2618{\cdot}4 \times 10{\cdot}472 = 27419{\cdot}8848$.

2. Suppose the length of the zone to be 60, its greatest and least diameters 40 and 30, and the central distance 20 inches. Required the content of the zone.

Ans. 64063·6178 cubic inches.

3. Suppose the length of the zone to be 48, its diameters 36 and 28, and the central distance 16 inches. Required the content of the zone. Ans. 42264·795495 cubic inches.

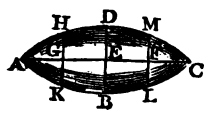
4. Suppose the length of the zone to be 30, its diameters 20 and 14, and the central distance 12 inches. Required the content of the zone. Ans. 7757·1034754 cubic inches

5. Suppose the length of the zone to be 36, its diameters 30 and 24, and the central distance 18 inches. Required the content of the zone. Ans. 22316·03429 cubic inches.

PROB. XV. To find the solid content of a parabolic spindle.

RULE. Multiply the square of the greatest diameter by the length and by ·7854, and $\frac{8}{15}$ of the product will give the content. Or multiply the square of the greatest diameter by ·418879 to get the content.

1. Suppose the length AC to be 80, and the greatest diameter BD 32 inches. Required the content.



Ans. $32^2 \times 80 \times ·7854 \times \frac{8}{15} = 81920 \times ·418879 =$ 34314·56768 cubic inches the content.

2. Suppose the length to be 64, and the greatest diameter 20 inches. Required the content. Ans. 10723·328 cu. in.

3. Suppose the length to be 84, and the greatest diameter 36 inches. Required the content. Ans. 45600·95232 cu. in.

4. Suppose the length to be 72, and the greatest diameter 42 inches. Required the content. Ans. 53200·984 cu. in.

5. Suppose the length to be 108, and the greatest diameter 38 inches. Required the content. Ans. 65325·017808 cu. in.

PROB. XVI. To find the content of the middle zone of a parabolic spindle.

RULE. To twice the square of the greatest diameter add the square of the least, and from the sum subtract $\frac{4}{10}$ of the square of the difference of these diameters; multiply the remainder by the length and by ·2618, to get the content.

1. Suppose the length FG to be 40, the greatest diameter BD 32, and the least HK 24 inches. Required the content.

Ans. $(32^2 \times 2 + 24^2) - \{(32 - 24)^2 \times ·4\} \times 40 \times ·2618$ $= 2624 \times 25·6 \times 48 \times ·2618 = 103936 \times ·2618 = 27210·4448$ cubic inches the content.

2. Suppose the length to be 42, and the diameters 34 and 27 inches. Required the content. Ans. 33222·10584 cu. in.

3. Suppose the length to be 48, and the diameters 36 and 30 inches. Required the content. Ans. 43700·91264 cu. in.

4. Suppose the length to be 44, and the diameters 34 and 28 inches. Required the content. Ans. 35497·56672 cu. in.

5. Suppose the length to be 38, and the diameters 30 and 24 inches. Required the content. Ans. 23494·14144 cu. in.

OF UNGULÆ.

PROB. I. To find the contents of the parts into which a frustum of a rectangular or square pyramid is cut, by a plane passing through one of the sides of the base.

RULE. One of the parts cut off will be a wedge, of which the content may be found by Prob. XI. MENSURATION OF SOLIDS; and this subtracted from the content of the whole will give the other part.

1. Let the perpendicular height of the frustum of a square pyramid be 287·9649 inches, and the sides of its bases 15 and 6 inches; and let a plane pass through one of the sides of the less base, and cut the side of the frustum at the perpendicular height of 119·98536 inches from that base: the length of the section it makes is 9·75 inches. Required the contents of the parts.

$\{(15+6)^2 - 15\times 6\}\times \frac{1}{3}\times 287{\cdot}9649 = 33691{\cdot}8933$ cu. in. the frustum.
$(12+9{\cdot}75)\times 6\times \frac{1}{6}\times 119{\cdot}98536 = 2609{\cdot}6816$ cu. in. the wedge.
Ans. 31082·2117 cu. in. the remaind.

2. Let the height of the frustum of a rectangular pyramid be 30 inches, the sides of the greater base 48 and 36, and those of the less base 36 and 27; and let a plane pass through the less side of the greater base, and cut the opposite at the height of 20 inches; the length of the section it makes with that side is 30 inches. Required the contents of the parts.

Ans. Wedge 15120, remainder 24840 cubic inches.

3. Required the contents of the parts of the frustum of a square pyramid, the sides of the bases 30 and 20, a plane through the greater base passes through the less base, the height 72 inches.

Ans. Wedge 28800, remainder 16800 cubic inches.

4. Required the contents of the parts of the frustum of a rectangular pyramid, the sides of the under base 40 and 30, and of the upper base 24 and 18, and the plane passes through the greater sides of the two bases, the height 42 inches.

Ans. Wedge 21840, remainder 11088 cubic inches.

5. Required the contents of the parts of the frustum of a rectangular pyramid, the height 60 inches, the sides of the under base 36 and 28, and of the upper 30 and 23⅓; a plane passes through the greater side of the lower base, and cuts the opposite side at the height of 30 inches; the section it makes is 33 inches.

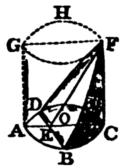
Ans. Wedge 14700, remainder 36260 cubic inches.

PROB. II. To find the content of the hoof of a cylinder.

RULE. Find the area of the base of the hoof, and multiply it by the difference between the radius and the versed sine or height of the base, and add the product to $\frac{1}{12}$ of the cube of the chord of the

base, if the height of the base be greater than the radius; otherwise subtract them: the sum or difference, multiplied by the height of the hoof, and divided by the height of the base, will give the content.*

NOTE. If the cutting plane pass through the centre of the base, multiply the square of the diameter by $\frac{1}{6}$ of the height of the hoof to get the content.



1. Suppose the diameter AC of the base of the cylinder to be 50, the height CF of the hoof 120, and the height or versed sine of its base CE 10 inches. Required the content of the hoof.

Ans. $10 \div 50 = \cdot200$ versine, of which the tabular area is $\cdot111823$, then $\cdot111823 \times 50^2 = 279\cdot5575$ circular segment, and $\sqrt{(40 \times 10)} \times 2 = 40$ the chord. Now $\{(40^3 \div 12) - (279\cdot5575 \times (40 - 25)\} \times 12 = 5333\cdot3333\dot{3} - 4193\cdot3625 \times 12 = 1139\cdot9708\dot{3} \times 12 = 13679\cdot65$ cubic inches the content.

2. Suppose the versed sine of the base to be 40, the rest as before. Required the content.

Here the chord is 40, the base 1683·9359. Ans. 91777·1875 cu. in.

3. Suppose the cutting plane to pass through the centre, the rest as before. Required the content. Ans. 50000 cubic inches.

4. Suppose the diameter of the cylinder 48, the versed sine of the hoof 30, and its height 36 inches. Required the content.

Ans. 18604·98 cubic inches.

5. Suppose the diameter of the cylinder 36, the height of the hoof 42, and its versed sine 12 inches. Required the content.

Ans. 5167·07117 cubic inches.

PROB. III. To find the content of the hoof of the frustum of a cone.

CASE I. When the cutting plane passes through the extremities of the two bases.

RULE. Take the square root of the product of the diameters at the base and the top of the hoof, and multiply it by the diameter at the top, then take the difference between this product and the square of the diameter of the base, and divide it by the difference of the diameters: the quotient, multiplied by the diameter of the base, by the height, and by ·2618, will give the content.

1. Suppose the diameter of the base of the hoof to be 30, and the diameter of the frustum at the top of the hoof to be 19·2, and the height 18 inches. Required the content.

Ans. $\sqrt{(19\cdot2 \times 30)} \times 19\cdot2 = 24 \times 19\cdot2 = 460\cdot8$, and $30^2 - 460\cdot8 \div (30 - 19\cdot2) = 439\cdot2 \div 10\cdot8 = 40\cdot\dot{6}$, then $40\cdot\dot{6} \times 30 \times 18 \times \cdot2618 = 21960 \times \cdot2618 = 5749\cdot128$ cubic inches the content.

2. Suppose the diameter at the base 19·2, that at the top 30, and the height 18 inches. Required the content.

Ans. 2943·553536 cubic inches.

3. Suppose the diameter of the base 24, the diameter of the top 18, and the height 36 inches. Required the content.

Ans. 7610·6089 cubic inches.

* For the demonstration of this and the following Problem see Appendix.

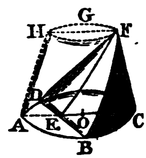
4. Suppose the diameter of the base 20, that at the top 28, and the height 14 inches. Required the content.

Ans. 2406·21259648 cubic inches.

5. Suppose the diameter of the base 15, that at the top 12, and the height 16 inches. Required the content. Ans. 1340·481136 cu. in.

CASE II. When the plane cuts off a part of the base.

RULE. Find the tabular area answering to the quotient of the height of the base by its diameter, and multiply it by the cube of that diameter for the first content. From the height of the base subtract the difference between the diameters at the top and the base of the hoof; take the tabular area answering to the quotient of the remainder divided by the diameter at the top, and multiply it by the cube of the diameter at the top, and by the quotient of the height of the base divided by the said remainder, and also by the square root of the same quotient, for another content. Multiply the difference of these contents by one-third of the height of the hoof; the product, divided by the difference of the diameters, will give the content of the under hoof; and this hoof, subtracted from the content of the frustum, will give the other hoof.



1. Suppose the height of the hoof to be 18, the diameter AC of the lower base 30, the diameter FH at the top 19·2, and that the plane cuts off CE 20 inches height from the lower base. Required the content.

The tabular area of $\frac{20}{30}$ is ·556226, which, multiplied by 27000, gives 15018·102 the first content; and the tabular area of $9{\cdot}2 \div 19{\cdot}2 = {\cdot}4791\frac{2}{3}$ is ·371872, which, multiplied by $19{\cdot}2^3$, and by $20 \div 9{\cdot}2$, and by $\sqrt{(20 \div 9{\cdot}2)}$, gives 8436·4657, which, subtracted from the former content, leaves 6581·6363; then this, multiplied by 6, and divided by 10·8, gives 3656·4646 cubic inches the content.

2. Suppose the plane to cut 15 inches for the height of the base, the rest as before. Required the content. Ans. 2517·8613 cu. in.

3. Suppose the height of the base 10·8 inches, the rest as before. Required the content. Ans. 1606·41 cubic inches.

NOTE. In this example, where the height of the base is equal to the difference of the diameters at the base and top, the tabular versed sine for the second area is nothing. Therefore, multiply the first tabular area by the cube of the diameter at the base, and divide the product by the height of the base, for the first content. Also, multiply the height of the base by the diameter at the top, and multiply the square root of the product by the same diameter, and to the product add one-third of itself, for the second content. The difference of these contents, multiplied by one-third of the height of the hoof, gives its content.

4. Suppose the diameter of the base 36, that at the top 27, the height 24, and the versed sine 18 inches. Required the content.

Ans. 4945·166936 cubic inches.

5. Suppose the diameter of the base 24, that at the top 32, the height 42, and the versed sine 16 inches. Required the content.

Ans. 11447·9264 cubic inches.

SURVEYING.

Surveying is the method of determining the magnitude, position, and shape of lines, fields, &c. on the surface of the earth.

For this purpose, various instruments are used for measuring lines and angles.

OF INSTRUMENTS USED FOR MEASURING LINES.

Straight lines are measured by applying to them a line of known length, as a foot, a yard, a chain, &c. a number of times.

The Chain used in surveying consists of 100 links, and is distinguished at the end of every 10 links by a small piece of brass cut into points to facilitate the counting of the odd links. Thus, at 10 links from either end the piece of brass has 1 point; at 20 links, it has 2 points; and so on to the middle of the chain, which is marked by a circular piece. Ten chains in length, and one in breadth, make an acre.

If the length of a pendulum vibrating seconds at Greenwich Observatory be taken 23 times, and the amount divided into 25 equal parts, each of these parts will be nearly an English yard, or 22 of them an English chain; therefore a link of it will be 7·92 inches.*

The Scotch chain was 74·1196 English feet long, and each link of it 8·89 inches. The Scotch ell was 37 Scotch inches, = 37·0598 English inches; and 6 ells made a fall.

The Offset-Staff is a pole of 10 links in length. It is divided into 10 parts, and the last of them subdivided into 10 smaller parts. Its use is for examining the chain, which is liable to stretch with long usage or the roughness of the ground. It is also used for measuring short distances, such as perpendiculars or offsets from the principal straight line to the enclosures.

The Cross consists of two pair of sights fixed on a pole, at

* The length of the pendulum vibrating seconds in a vacuum at the level of the sea in the latitude of London is 39·1393 inches; 23 times the length of this pendulum is = 900·204 inches; and 25 yards = 900 inches; so that 23 times the length of the pendulum is only about ⅕ of an inch more than 25 yards.

right angles to one another. Its use is to determine the point in which a perpendicular from a corner would meet the principal line that is measured. It is moved backwards or forwards along the line, keeping its extremities in view through one pair of the sights, till the corner from which the perpendicular comes is seen through the other pair of sights: the cross is then at the foot of the perpendicular.

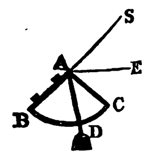
The PERAMBULATOR is sometimes used for measuring roads, &c. It turns upon a wheel, of which the circumference is 8·25 feet; so that 8 revolutions make an English chain in length. The distance measured is pointed out by an index moved by machinery. As, however, by its entering into hollows, and going over small eminences, it must give the distance too great, much reliance cannot be placed on the result.

OF THE INSTRUMENTS USED FOR TAKING ANGLES.

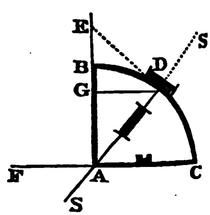
Angles in the field are taken either in a vertical or in a horizontal plane. The former are measured by a Quadrant, and the latter by a Theodolite or Circle.

A QUADRANT is the fourth part of a circle of any convenient radius. It is made of brass or wood, and the arc is divided into 90 degrees, and each degree is subdivided into smaller parts. The degrees are numbered from one extremity, called the beginning of the arc, to the other extremity or end of it.

The most simple quadrant, ABC, has a line with a plummet suspended from its centre, as AD, which, when hanging freely, is always perpendicular to the horizon; and *sights*, or a telescope, is affixed to the radius AB, which passes through the 90th degree, or end of the arc, to direct the eye in a straight line towards the object.



Sometimes an index AD, with telescopic sights, is made to revolve round the centre A; in which case a spirit-level is fixed to the radius AC, which passes through the beginning of the arc. The telescope is placed along AD. But sometimes the degrees are numbered from B, and a telescope is fixed at D, perpendicular to the index AD.



The THEODOLITE is the most complete instrument for surveying. It consists of a circular brass plate, the circumference of which is divided into 360 degrees, or twice 180 degrees,

and each degree is subdivided into smaller parts. An index with a compass on it is fixed to the centre, and revolves round it; and on it is erected a semicircle, perpendicular to the plane of the instrument, furnished with a telescope perpendicular to the index of it, which moves round its centre. The use of the circle is for taking horizontal angles, and that of the semicircle is for taking vertical ones. The instrument is furnished with two spirit-levels for placing the plate, and the telescope, when at the top of the semicircle, in a horizontal direction; in subservience to which, the tripod upon which the instrument stands has four screws, &c. A more particular description of this instrument, in its most improved state, would scarcely be intelligible to a learner, without seeing and using it; and it is therefore omitted here.

The CIRCUMFERENTER is a circle, on the centre of which is a large compass; and the circumference is divided, not only into points and quarters, but also into degrees and parts of a degree. An index or two is moveable about the centre. Its use is the same with that of the theodolite; only, when using it, greater reliance is placed upon the compass. It is chiefly used for surveying mines, or large tracts of land where great accuracy is not required.

Large LEVELS, with telescopic sights, are often requisite for finding the elevation of one place above another. And the surveyor ought also to be possessed of several pocket-levels, to be applied when occasion requires them.

Each of the indices of these instruments has a NONIUS, for enabling the surveyor to read off minutes. The nonius is a scale on which the number of divisions is greater by one than the number in the same space upon the arc. If the nonius occupy the space of 29 divisions on the arc, it is divided into 30 equal parts, by which means each division will exceed one on the nonius by $\frac{1}{30}$ of a division on the arc; so that, by moving forward the index $\frac{1}{30}$ of a division of the arc, the first one on the nonius will coincide with one on the arc; and by moving another $\frac{1}{30}$, the second will coincide, and so on. Consequently, if the arc be divided into half degrees, the nonius will point out minutes.

The PLANE-TABLE is an instrument much used in surveying, when the survey is not large, because it gives the plan of the ground, as well as its quantity. It is a rectangular board fixed upon a tripod, with a ball and socket for giving it any inclination. It has a loose frame fitted to it, one side of which is divided into equal parts all around; and the other side is divided into 360 degrees, by lines directed to the centre of the table; and a compass is fastened to one of the sides of the table.

There is a loose index to be used with it, having a telescope placed parallel to its fiducial side; and there are several plane scales upon the index, for laying down the measured distances. A sheet of paper, moistened equally with a sponge, is spread upon the table, and the frame pressed down upon it to keep it fixed. The paper will become smooth when it is dry, and it will then be fit for drawing the plan upon.

An angle may be measured with the plane-table, by placing that side of the frame uppermost which has degrees on it, and proceeding as with the theodolite. Or the angle may be drawn on the table, by directing the index to marks in the sides of the angle in the field; and, in like manner, a given angle may be formed in the field by the table. Also, a perpendicular may be drawn in the field with it, by placing the centre of the instrument at the given point, and turning it, till the index, while cutting the same divisions on opposite sides of the frame, is in the direction of the given line: then, if the index be made to cut similar divisions on the other sides of the table, it will give the direction of the perpendicular.

OF SHIFTING THE PAPER ON THE PLANE-TABLE.

When one paper is full, and there is occasion for more, draw a line in any manner through the farthest point of the last station-line, to which the work can be conveniently laid down; then take the sheet off the table and fix on another, drawing a line on it, in the most convenient part for the rest of the work; then fold or cut the sheet formerly used by the line drawn on it, apply the edge to the line on the new sheet, and, as they lie in that position, continue the last station-line on the new paper, placing on it the rest of the measure, beginning at the point where the previous sheet left off.

When the work is finished, the different sheets used must be carefully joined, so that the lines may come together in the same manner as when the lines were transferred from the old sheets to the new ones.

It may be noticed, that if the joining lines on the old and new sheets have not the same inclination to the side of the table, the needle will not point to the original degree when the table is rectified; and if the needle be required to point still to the same degree of the compass, the easiest way of drawing the lines in the same position is to draw them both parallel to the same sides of the table, by means of the equal divisions marked on the other two sides.

INSTRUMENTS USED IN DRAWING PLANS.

The surveyor ought to be provided with compasses of various sizes, some of which must have very fine points, both

of steel and for ink. He ought also to have drawing-pens of different finenesses, for drawing coarse and fine lines; and a number of scales of various sizes, from one chain in an inch to 8 or 10 chains in an inch, which ought to have the divisions marked on the edges for laying down distances without compasses. He will also stand in need of lines of chords, and protractors of different radii; and, for the sake of expedition, he ought to use parallel and perpendicular rulers and reducing scales.

PROB. I. To measure a straight line in the field.

Erect poles at the extremities, and at convenient distances along the line, for showing the direction. Ten arrows of iron or wood are used for marking the spot to which the chain extends, and for preserving the number of chains. Let the leader, or the person going before, take the end of the chain and the ten arrows; and having stretched the chain, and taken notice that none of the links are involved in one another, let the follower, placing the end of the chain at the extremity of the line, direct him, by waving his hand towards the right or left, into the proper direction. And the leader having fixed an arrow at the end of the chain, let them both go forward with the chain, till the follower comes to the arrow: there let him direct the leader as before, who fixes another arrow, while the follower takes up the former one. Let them proceed thus, till all the arrows are in the hand of the follower, and the chain stretched beyond the last of them; then let the arrows be conveyed to the leader, and let him fix one of them at the end of the chain, and proceed in the same way till all the arrows are again changed, or till he has arrived at the end of the line to be measured. And at the last, let the follower reckon the number of changes, the number of arrows in his hand, and the number of links between him and the extremity of the line. Thus, 3 changes 7 arrows and 45 links, make the length of the line to be 3745 links.

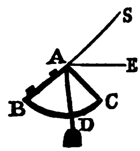
NOTE 1. The surveyor, while measuring a straight line, ought carefully to take notice of every surrounding object of which the position can be more easily determined from it than from any other line which he intends to measure. He ought to mark the distance at which the line meets a corner, or crosses a boundary, or begins or ceases to run along a hedge, a wall, or a road. He must likewise mark the distances at which perpendiculars or offsets are to be raised; and, in general, every thing which may tend to shorten his other operations in the survey, or will assist him in drawing his plans. When he has settled, by the cross or otherwise, the place of an offset or short perpendicular, it will be easiest to measure the length

of it as he goes along, to save the time and trouble of returning to the place a second time.

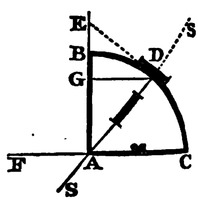
NOTE 2. The plan ought to be drawn upon paper, with horizontal distances only; otherwise it will be impossible to join several fields together without distortion. For when several lines are to be joined together, a small error in the lengths of some of them will alter the position of others; a circumstance which has a greater tendency to distort the plan, than even the lengths of the lines themselves. It is, however, impossible for a surveyor to ascertain the exact level of every elevation and depression of his lines; but it would be of great advantage to him to take a level at that part which he judges to have a mean inclination. This may be done with the offset-staff thus:—Having laid the chain along that part, place one end of the offset-set staff at the uppermost of 10 links on it, and let the assistant take the other end, and a line and plummet hung exactly over the other end of the 10 links on the chain, and let the surveyor apply a pocket or other level to the staff; and when it is level, the line of the plummet will point out on the staff the horizontal length of the 10 links of the chain. Consequently, by using a diagonal scale of 10 to a link, it will point out how much the line is to be diminished to get the horizontal length of it.

PROB. II. To take a vertical angle in the field.

Vertical angles are denominated Angles of Elevation when the object is higher than the eye, and Angles of Depression when it is lower.



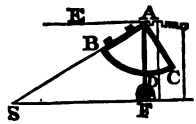
1. *To take an angle of elevation.* If the quadrant ABC have a plummet, place the eye to the limb B, and look through the sights in AB to the object S, and the line and plummet AD hanging freely, will cut off the arc CD from the end C, farthest from the sights, the degrees, &c. of which will be the measure of the angle EAS, contained by the horizontal line AE, and the visual ray AS; for DAE and CAS are right angles.



If the quadrant have a telescope fixed on the index AD, which moves about the centre A: Having levelled the radius AC, and directed the quadrant towards the object S, move the index AD till S is seen at the crossing of the wires of the telescope; then the arc CD is the measure of the angle CAD.

If the telescope be at D, perpendicular to AD, move the index, till, looking through the telescope, the object E is in the centre of the telescope; then the arc BD is the measure of the angle of elevation.

2. *To take an angle of depression.* If the quadrant ABC have a plummet, place the eye at the centre A, and look through the sights in the radius AB to the object S below, and the line of the plummet AD will cut off the arc CD, the measure of the angle of depression EAS; for EAD and BAC are right angles.



If the telescope be on the index AD, place the eye at the limb D, and look down to S through the telescope; and the arc CD is the measure of the angle of depression.

If the telescope be perpendicular to the index, depress the object-glass till the object be seen; and the arc BD between the index and the vertex is the measure of the angle of depression.

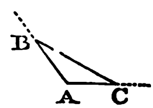
PROB. III. To measure a horizontal angle in the field.

WITH THE THEODOLITE. Having placed the instrument at the angular point, and the cipher of the index at the beginning of the degrees on the circle, turn the whole instrument about till a distant pole in one of the sides of the angle be seen in the centre of the telescope; there fix the instrument, and turn the index upon it, till a pole fixed in the other side of the angle be seen in the centre of the telescope: then the degrees, &c. moved over by the index is the measure of the angle.

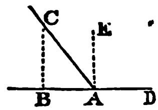
WITH THE CIRCUMFERENTER OR THE COMPASS. Having fixed the instrument, so that the north point of the compass point to the fleur-de-lis, direct the sights to a mark in one side of the angle, and mark the degrees, &c. pointed out by the needle. Then turn the sights towards a mark in the other side of the angle, and again mark the degrees cut by the needle. Their sum or difference, according as they are on different or on the same side of the north or south points, will give the quantity of the angle.

NOTE. The degrees marked show the bearing of the sides of the angle, allowance being made for the variation.

WITH THE CHAIN. Extend the chain along one of the sides, from the angular point A to B, and along the other side from A to C, and measure from C to B. Then, having drawn the triangle ABC upon paper, the angle BAC may be measured with a protractor, or with the line of chords.



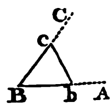
NOTE. If a table of natural sines be at hand, look among the sines for ½BC, and the degrees, &c. answering to it will be half the angle BAC.



WITH THE CROSS. If the angle be acute, as BAC, place the cross at B in one of the sides of the angle, so that one pair of the sights may be directed along AB; and, looking through the other pair of sights, let an assistant mark the point C of the line AC, which is seen through them; and then the angle BAC is determined by measuring AB and BC. If the angle be obtuse, as CAD, it may be determined by measuring its supplement BAC, or by placing the cross at A, so that AD may be seen through one pair of the sights; then let an assistant place a distant mark at E, seen through the other pair of sights; after which measure the angle EAC as before, and add a right angle to it.

PROB. IV. To make or lay down an angle in the field.

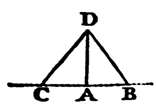
WITH THE THEODOLITE. Having placed the instrument at the point at which the angle is to be made, and fixed the index at the beginning of the degrees, turn the theodolite until a mark is seen in the given line; there fix it, and turn the index upon it, the proper way, over the given number of degrees; then, looking through the telescope, direct an assistant to place a mark.



WITH THE CHAIN. The angle must first be made on paper, as ABC. Make B*b* and B*c* each 30, and measure *bc*. Lay 30 links on the given line on the ground from B to *b*; and having reckoned as many links of the chain as are in the sum of B*c* and *cb*, fix the ends of them at B and *b*, and, taking 30 links from B in your hand, go backward till both ends of the chain are equally stretched, and there fix a pin in the ground, which will give *c*.

PROB. V. To raise a perpendicular in the field.

WITH THE THEODOLITE, CIRCUMFERENTER, &c. At the given point in the line make an angle of 90°, by the last problem.

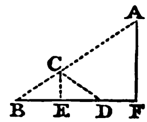


WITH THE CROSS. Having placed the cross at A, and directed one pair of the sights to a mark B in the given line, look through the other pair of sights, and cause a mark D to be placed in that direction.

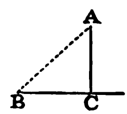
WITH THE CHAIN. Measure in the given line 30 links from A to B, and as many from A to C; and, fixing the ends of the chain at B and C, take hold of the 50th link, and go backwards till both ends of the chain are equally stretched, and there fix a pin at D; AD will be perpendicular to BC.

PROB. VI. To drop a perpendicular in the field.

WITH THE CROSS. Move the cross along the given line, so that its extremities appear through one pair of the sights, until the given point is seen through the other pair. The instrument is then in the point of the line upon which the perpendicular falls.

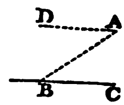


WITH THE CHAIN. Measure a straight line from the point A to any point B of the given line. Let BC be a chain in that direction. Fix one end of the chain at C, and with the other go along the given line till the chain is again stretched, and there make a mark, as at D. Measure BD, and multiply ½BD by BA, and cut off two figures from the right of the product: the rest will give BF, the distance of B from the foot of the perpendicular AF.



WITH THE THEODOLITE. Fix the instrument at any point B of the given line BC, and measure the angle ABC (by Prob. III.); then fix the instrument at A, and (by Prob. IV.) make the angle BAC the complement of ABC, and AC will be the perpendicular required.

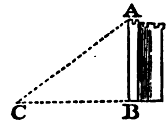
PROB. VII. To run a line in the field parallel to a given straight line BC.



Take any point B in the given line BC, and measure the angle ABC contained by BC, and the line directed to the given point A; then at A make the angle BAD equal to ABC, and AD will be the direction of the parallel.

OF HEIGHTS AND DISTANCES.

PROB. VIII. To find the height of an object A, when the point B on the level ground, directly below it, is accessible.



On the level ground measure any distance BC in a straight line, and at C take the angle of elevation ACB with a quadrant. Then rad. : tan. C : : CB : BA; and if CA be required, then sin. C : R : : BA : AC (Theor. I. Trig.)

1. In the triangle ABC, right-angled at B, are given BC 236 feet and the angle ACB 35° 48′. To find AB.

C 35° 48′ tan. — R	$\bar{1}$·858069	BA 170·208	log. + R	12·230981
CB 236 feet log.	2·372912	C 35° 48′	sine	9·767124
Height BA 170·208 feet log.	2·230981	AC 290·976 feet	log.	2·463857

Note. The height thus obtained is that above the level of the eye of the observer, and must be increased by the height of the eye, to have its height above the level ground. The same is to be done in all the observations on heights.

2. From the bottom of a steeple I measured upon a level plane a straight line 136 feet, and at its extremity I took the elevation of the top of the steeple 47° 25′. Required the height of the steeple. Ans. 147·985 feet.

3. The elevation of a wall, taken from the edge of the ditch 18 feet wide, was 62° 40′. Required the height of the wall, and the length of a ladder to reach the top of it.

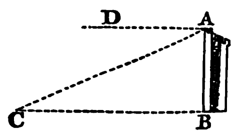
Ans. Height 34·8246, ladder 39·20153 feet.

4. At 85 feet from the bottom of a tower, the angle of its elevation was 52° 30′. Required its altitude.

Ans. 110·774 feet

5. Near the bottom of a hill I took the elevation of its top 54° 40′, and the altitude of the hill was 1156 feet. Required the distance of my station from its top. Ans. 1417·0127 ft.

Prob. IX. From the top of a known height AB, to find the distance of an object C, on the plane below.



Take the angle of depression CAD; then, in the triangle ABC, right-angled at B, are given AB, and the angle ACB = DAC. Then sin. C : R : : AB : AC, and if the horizontal distance CB be required, tan. C : R : : AB : BC (Theor. I. Trig.)

Note. If AC be given, then R : cos. C : : AC : CB, and R : cos. A : : AC : AB (Theor. I. Cor. 1. Trig.)

1. Suppose AB 83 feet, and the angle ACB 23° 37′, required AC and CB.

AB 83 log. + R	11·919078	83 log. + R		11·919078
C 23° 37′ sin.	9·602728	23° 37′	tan.	9·640716
AC 207·181 log.	2·316350	BC 189·829	log.	2·278362

2. Let the sloping side of a hill AC be 268 feet, and the angle of depression at its top DAC be 33° 45′. Required the base BC, and its particular height AB.

Ans. BC 222·834, AB 148·893 feet.

3. From the top of a mast 80 feet high the angle of de-

pression of another ship's hull was 20°. Required their distance. Ans. 219·798 feet.

4. From the top of a tower 120 feet high I took the depression of two trees 57° and 25° 30′. Required their distances from the tower and from each other.

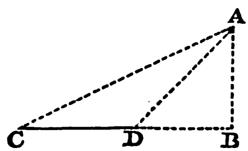
Ans. 77·93 feet, and 251·58 feet, and 173·65 feet.

5. Suppose the mean semidiameter of the sun subtends at the earth an angle of 16′ $7\frac{1}{2}''$; what is his distance from the earth? Ans. 213·1946 semidiameters.

6. From the top of a lighthouse 110 feet high I observed two ships in a straight line from it, and took the angles of depression of their hulls 56° 44′ and 18° 26′. Required their distance from the lighthouse.

Ans. 72·1649 feet, and 330·031 feet.

PROB. X. To measure an inaccessible height AB.



On the level ground measure any distance CD, in a straight line towards the height, and at C and D take the angles of elevation ACB and ADB; their difference is CAD. Then sin. CAD : sin. ACD : : CD : DA (Theor. II. Trig.) and R : sin. ADB : : DA : AB (Theor. I. Trig.) That is, sin. C × sin. D × CD ÷ sin. (C — D) = AB.

Or the difference of the natural cotangents of C and D is to the radius as CD to AB.

1. Let CD be 248 yards, the angles ACB 23° 30′, and ADB 37° 24′; then CAD is 13° 54′.

37° 24′	sine 9·783458	Nat. cot. 1·307946		
23° 30′	sine 9·600700	Nat. cot. 2·299843		
Dist. 248 log.	2·394452	Diff. 0·991897	log.	9·996466
	21·778610	248 log. + R.		12·394452
13° 54′ sin. + R.	19·380624	AB 250·0264	log.	2·397986
AB 250·0264 log.	2·397986			

2. Sailing in a boat, a hill was observed, and the elevation of its top above the level of the sea was 27° 38′. After sailing 540 fathoms, each 5 feet, directly towards the hill, the elevation of its top was 35° 28′. Required the height of the hill above the level of the sea. Ans. 1066·268 fathoms.

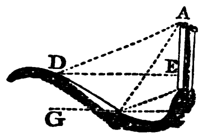
3. The elevation of a hill at the bottom of it was 46°, and at 100 yards distance 31°. Required the height of it.

Ans. 143·1452 yards.

4. The angle of elevation of a tower was 26° 30′, and, 75 yards nearer to it, the elevation was 51° 30′. Required its height and distance. Ans. Height 61·97, dist. 49·2934 yds.

5. Measured 149 yards towards a hill, and at the extremities of the line the elevations of its top were 29° 17′ and 39° 25′. Required its height. Ans. 263·02 yards.

PROB. XI. To measure a height which has no level ground before it.



Take two stations C and D, in a vertical plane, and measure CD; at C take the elevation of D above C, or the angle GCD, and the elevations or depressions of the top and bottom of the height, viz. the angles ACF and BCF; at D take the elevation of the top, or the angle ADE. Since the angle EDC = DCG; therefore ADC = ADE + DCG and DAC = ACF — ADE. Hence the triangle ADC has two angles, ADC and DAC, and the side CD given to find the side AC. Then in the triangle ACB are given the angles ACB = ACF ± BCF, and ABC = 90° ± BCF, and the side AC to find the side AB; wherefore sin. DAC : sin. ADC : : DC : CA, and sin. ABC : sin. ACB : : CA : AB (Theor. II. Trig.)

1. Suppose the angles GCD 31° 26′, ACF 53° 26′, BCF 18° 32′, and ADE 22° 30′, and the distance CD 286 feet. Required the height AB.

Hence the angle ADC = (22° 30′ + 31° 26′) = 53° 56′ DAC = (53° 26′ — 22° 30′) = 30° 56′, and ACB = (53° 26′ ± 18° 32′) = in this case (since F is below B) 34° 54′; then

ADC 53° 56′		sine 9·907590
DC 286		log. 2·456366
ACB 34° 54′		sine 9·757507
DAC 30 56	ar. co.	sine 0·289003
ABC 71 28	ar. co.	sine 0·023128
AB 271·39 ft.		log. 2·433594

NOTE 1. If DE be above A, the angle DAC is the sum of ACF and ADE; otherwise it is their difference. Also, in this case ADC is the difference of DCG and ADE; otherwise it is their sum. Also, when F is below B, the angle ACB is the difference of ACF and BCF; otherwise it is their sum.

NOTE 2. If the stations C and D cannot be conveniently taken in a vertical plane, they may be taken anywhere, and then the angles ADC and ACD must be measured with a sextant, and the triangle ACD will give the side AC.

2. At a considerable distance from a hill, I took the elevation of the top of a tower built upon it, 33° 45′; and measuring on level ground 300 feet directly towards the hill, I

again took the elevations of the top and the bottom of the tower 51° and 40°. Required the height of the tower.

Ans. 46·666 yards.

3. At a window on a level with the base of a steeple I took the elevation of its top 40°; and at another window of the same house, 18 feet higher, I took again the elevation of the top of the steeple 37° 30′. Required the height of the steeple.

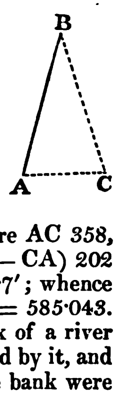
Ans. 210·44 feet.

4. The elevation of the top of a hill at one station was 38° 25′. Another station was taken 450 feet from the first, but neither on a level with it nor in the direction of the hill. At the first station, the line from the other station to the top of the hill subtended an angle of 67° 30′; and at the second, the line from the first to the top of the hill subtended an angle of 74° 48′. Required the height of the hill. Ans. 441·25 feet.

5. I measured directly up a hill 132 yards: there I took the depression of the hill 42°, that of the bottom of a distant object 27°, and that of its top 19°. Required the height of the object. Ans. 28·6367 yards.

Prob. XII. To find the distance of a place A, from an inaccessible object B.

When B is visible from A.



Choose a station C, from which both A and B can be seen. Measure AC, 650 yards, and take the angles BAC 72° 22′, and ACB 78° 37′, with the theodolite. Then ABC is 29° 1′, and sin. B : sin. C : : CA : AB = 1313·67 yards.

When B is not visible from A.

Choose a station C from which both A and B may be seen, and their distances from it measured. Take the angle ACB 75° 38′, and measure AC 358, and CB 560 feet. Then (BC + CA) 918 : (BC — CA) 202 : : tan. $\frac{1}{2}$(A + B) 52° 11′ : tan. $\frac{1}{2}$(A — B) 15° 49·7′; whence BAC is 68° 0·7′, and sin. A : sin. C : : CB : BA = 585·043.

3. A straight line was measured along the bank of a river 528 feet, and at its extremities the angles contained by it, and straight lines directed to a tree upon the opposite bank were 62° 40′ and 73° 26′. Required the breadth of the river.

Ans. 676·445 feet to the nearest station, and 648·366 perp. breadth.

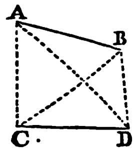
4. Straight lines from a station to two places measured 694 and 456 yards, and the angle contained by them was 127° 16′. Required the distance of the one place from the other.

Ans. 1035·772 yards.

5. To find the distance between two trees, I found the angle

it subtended at a station to be 55° 40′, and measured from the station to the trees 588 and 672 yards. Required their distance. Ans. 592·97 yards.

PROB. XIII. To find the distance between two places, both of them inaccessible.



1. To find the distance of two places A and B, on the opposite side of a river, I took two stations, C and D, distant 1267 links from one another, and such, that from each of them the other station and the places A and B were seen. At C I took the angles BCA 53° 38′, and BCD 34° 50′, and at D the angles ADC 43° 44′, and ADB 58° 38′. Required the distance between A and B.

In the triangle ADC, the angle ACD is 88° 28′, and CAD 47° 48′; hence sin. A : sin. C : : CD : DA = 1709·69. In the triangle BCD, the angle CDB is 102° 22′, and CBD 42° 48′; hence sin. B : sin. C : : CD : DB = 1065·14. In the triangle ADB are given AD and DB, and the angle ADB; therefore (AD + DB) 2774·83 : (AD — DB) 644·55 : : tan. $\frac{1}{2}$(A + B) 60° 41′ : tan. $\frac{1}{2}$(A — B) = 22° 28$\frac{1}{3}$′; whence ABD is 83° 9$\frac{1}{3}$′, and sin. ABD : sin. ADB : : DA : AB = 1470·3 links.

2. To find the distance between two steeples A and B, I took two stations C and D, distant 428 yards from one another; and at C took the angles ACB 54° 30′, and BCD 42° 26′; and at D took the angles CDA 40° 44′, and ADB 57° 42′. Required the distance of the steeples. Ans. 546·7 yards.

3. To find the distance between two places M and P, I took two stations A and B, distant from one another 908·36 feet; and at A took the angles PAM 14° 34′, and MAB 46° 16′; and at B took the angles ABP 96° 44′, and PBM 18° 39′. Required the distance between M and P. Ans. 674·6375 ft.

NOTE. If the distance between the objects be known, and the distance between the stations be required, assume 1 or 1000 for the distance between the stations, and with it find the distance between the objects. Then, as the distance found is to the given distance, so is 1000 to the true distance between the stations.

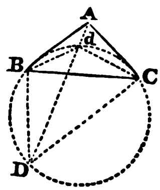
4. Suppose the distance AB 700 feet, and at the station C let the angles ACB be 42° 45′, and BCD 54° 12′, and let the angles at D be ADB 50° 19′, and ADC 57° 33′. Required the distance CD. Ans. 330·04 feet.

5. To find the distance between two lighthouses A and B, I measured the distance between two stations M and R, 3370 yards, and at M took the angles AMB 37° 52′, and BMR

91° 27′, and at R the angles ARM 29° 56′, and ARB 40° 27′. Required the distance AB. Ans. 7063·36 yards.

6. At a station C, I took the angle ACB, subtending a line AB 3291 yards, and found it 4° 35′, and the angle BCD between B and another station D 86° 52′; and at D took the angles ADB 8° 24′, and ADC 70° 23′. Required the distance of the stations from one another. Ans. 3370·248 yards.

Prob. XIV. Given the distances of three places, A, B, C, from one another, viz. AB 317, AC 308, and BC 478 feet, and the angles which these distances subtend at a station D in the same plane with them, viz. ADB 24° 50′, and ADC 27° 44′; to find the distance of the station D from each of the places.



Having drawn the triangle ABC, make at the point C, on the side of BC, opposite to that on which the station D lies, the angle BCd 24° 50′, and at B the angle CBd 27° 44′, and about the triangle BCd describe a circle, and join Ad, meeting the circle again in D, and join BD and DC.

The three sides of the triangle ABC are given to find the angle ABC = 39° 25′ 14·6″; then ABd = ABC ± dBC = 67° 9′ 14″, when A and d are on different sides of BC, or = 11° 41′ 14·6″, when, as here, A and d are on the same side of BC. Also, the angles of the triangle BCd are given, with the side BC, to find Bd = 252·7 feet. Again, in the triangle ABd are given the sides AB and Bd, and the included angle ABd, to find the angles AdB = 131° 53′ 53″, and BAd = 36° 24′ 53″. Then in the triangle ABD are given the angles and the side AB, to find BD = 448·066, and AD = 661·738. And in the triangle DBC are given the angles and BC, to find DC = 591·563 feet.

2. If A be the place nearest to D, the angle BAd is 46° 47′ 32·2″; then BD is 550·153, AD 282·25, and CD 528·4 feet.

Note 1. If the given station be within the triangle, as at d, make the angles BCD and CBD equal to the supplements of BdA and AdC.

Note 2. If two of the given places, A and B, be in a straight line with the station D, the distances BC and CA subtend the same angle BDC. After finding the angle at B, work the triangle DBC.

Note 3. If the three places A, B, C, be in a straight line, the first operation will not be required. The rest are the same as before.

3. The three sides of the triangle ABC are AB 280, BC 314, and AC 326 yards; and from the station D without the triangle, the angle ADB was 25° 52′, and ADC 23° 6′, the

point C being the nearest to D. Required their distances from D. Ans. AD 586·163, BD 413·4114, CD 308·1078 yds.

4. Suppose AB 267 feet, BC 209, and AC 346, and at the point D, within the triangle, the angle ADC is 128° 40′, and ADB 91° 20′. Required the distances of D from the angles.

Ans. AD 195·357, BD 85·98, and CD 188·5074 feet.

NOTE. When D is in one of the sides, describe a segment on BC containing the given angle.

5. Suppose AB 122, BC 74, and AC 82 chains, and at D in AB, produced beyond B, the angle ADC is 22° 45′. Required the distance of D from the angles.

Ans. AD 181·79, BD 59·79, and CD 125·434 chains.

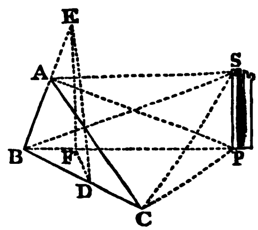
6. Suppose AB 1234, BC 873, and AC 632 yards, and at D in AB the angle ADC is 120°. Required its distance from the angles.

Ans. AD 226·117, BD 1007·883, and CD 487·84 yards.

7. Suppose AB 138, BC 224, and AC 326, and at D the angles are ADB 7° 22′, and ADC 19° 58′. Required the distance of D from the angles.

Ans. AD 510·9635, BD 385·2876, and DC 204·875.

PROB. XV. Given the angles of elevation of a tower PS, taken at three stations A, B, and C, on a level plane, no two of which are in the same vertical plane with the tower, viz. PAS 20° 10′, PBS 18° 50′, and PCS 34° 30′, and also the distances between the stations AB 324, BC 568, and AC 672 yards; to find the height of the tower.



Make the triangle ABC, of which AB is 324, BC 568, and AC 672 yards; make BE = BC, and BD = BA. Join ED, and upon it make the triangle EDF on either side of DE, so that BE : EF : : cot. PBS : cot. PAS and BD : DF : : cot. PBS : cot. PCS; or make EF = 527·495, DF = 160·792, and join BF, and make the angle BAP = BFE. Then erect PS perpendicular to the plane ABP, and in the plane passing through AP and PS make the angle PAS = 20° 10′, and PS will be the tower required.

Join PC, CS, BS, the triangles APB, FBE, being similar, AP : PB : : FE : EB : : cot. SAP : cot. SBP, therefore SBP is 18° 50′; also PB : BE = BC : : BA = BD : BF, there-

fore the triangles PBC and FBD are similar; and BP : PC : : BD : DF : : cot. PBS : cot. PCS, therefore PCS is 34° 30′.

In each of the triangles EBD, EFD, are given the three sides, to find the angles BED 28° 45′ 31″, and FED 6° 47′ 24″; then their difference 21° 58′ 7″, or their sum 35° 32′ 55″, is the angle BEF, from which, with the sides BE and EF, the angle BFE or BAP is found in the first case to be 89° 48′ 3″·7, and in the other 78° 48′ 18″·2. Therefore AP is 804·313 or 507·692, and PS is 295·3986 or 186·4592.

2. Let AB be 326, BC 584, and AC 683, and the angles of elevation SAP 30°, SBP 26°, and SCP 23°; to find PS.

Ans. PS is 952·161 or 168·645.

3. Let AB be 80, BC 119, and AC 140 feet, and the elevation at A 50°, at B 60°, and at C 55°. Required the height of the object D. Ans. 305·431 or 97·3602 feet.

4. Let AB be 60, BC 72, and AC 132 feet, and the elevations of S at A 30° 48′, at B 40° 33′, and at C 50° 23′. Required the height of S. Ans. 94·8328 feet.

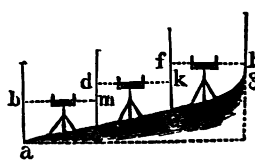
5. Let AB and BC be each 84 feet, and the points A, B, C, in a straight line, and the elevation at A 36° 50′, at B 21° 24′, and at C 14°. Required the height of the object.

Ans. 53·9606 feet.

OF LEVELLING.

When the altitudes of the several parts of an irregular ascent are to be determined, the surveyor should be provided with a Spirit Level, with telescopic sights, and one or two square poles, which slide out to the length of 20 or 25 feet, divided into feet and hundredth parts of a foot. On each pole is fitted a moveable vane, with a strong black line drawn horizontally between two white ones. A small level is also fixed upon the top of the under-part of the pole, to assist in holding it perpendicular during the observations.

Prob XVI. To find the height of g above a.



Place an assistant with the pole ab at a, and another with the pole cd at c, and having fixed the level nearly midway between them, turn the telescope towards a, and direct the assistant to move the vane upwards and downwards upon the pole till the black line on it coincide with the horizontal hair in the telescope, and then let him fix the vane. The feet and hundredth parts of a foot, cut by the under-part

of the vane upon the pole, are then carefully read off, and entered into the surveyor's book. The telescope is then turned towards the pole at *c*, and the assistant is directed, the height read off, and entered as at the first station. The pole at *a* is now removed and placed at *e*, whilst that at *c* still remains; the level is again placed in the middle between them, and the observations made and registered as before; and so on till the whole is finished. The difference between the sums of the heights of the back-observations, or those taken with the telescope directed towards *a*, and that of the fore-observations, or those taken towards *g*, will show the height of *g* above *a*.*

To find the height of any point *c* in a regular ascent: The distance *ag* is to *ac* as the height of *g* above *a* to the height which *c* ought to have above *a*.

It is not necessary to place the poles in the same direction with *ab* and *gh*, but it is necessary to erect them perpendicular, or nearly so.

NOTE. When the distance between the poles *ab* and *cd* is very great, the line *bm* will differ a little from the true level; for *bm* is a tangent to a great circle of the earth, passing through the centre of the instrument, and the true level is the arc of that circle between the poles *ab* and *od*. The correction may be neglected when the distance between the stations does not exceed 300 or 400 yards, and the instrument is placed in the middle between them: for a mile it is 7·96 or 8 inches; and for other distances from the instrument, the correction varies as the square of the distance.

1. To determine the height of an eminence, the following observations were taken :—

No.	Back.	Fore.	Ascent. Feet.	No.	Back.	Fore.	Ascent. Feet.
1	2·174	8·216	6·042	7	11·273	2·756	82·081
2	1·276	11·127	15·893	8	2·184	25·763	105·660
3	3·111	18·713	31·495	9	0·516	24·738	129·882
4	2·756	21·847	50·586	10	0·213	23·716	153·385
5	4·210	20·175	66·551	11	3·276	20·516	170·625
6	0·314	24·361	90·598	12	2·143	15·726	184·208

* Two poles are not necessary, for, after taking the back-observation upon the pole at *a*, it may be removed to *c*, and the fore-observation taken; then, removing the level into the second position, another back-observation is taken, and the pole removed to *e* for another fore-observation, and so on.

2. Determination of the height of Carnethy Hill, one of the highest peaks in the Pentland Chain, above the waste-wear of the Compensation Pond, by the method of Levelling.*

No.	Back.	Fore.	No.	Back.	Fore.	No.	Back.	Fore.
1	6·158	8·942	20	0·267	23·084	39	0·757	23·799
2	0·587	21·460	21	1·207	22·977	40	0·942	23·797
3	0·892	22·246	22	0·498	23·791	41	0·242	23·919
4	1·599	22·090	23	0·875	23·195	42	1·111	23·900
5	1·145	21·998	24	0·255	23·635	43	1·245	23·359
6	0·609	23·381	25	1·346	23·467	44	1·503	22·917
7	1·902	22·874	26	0·553	23·827	45	0·512	23·404
8	1·874	23·860	27	0·579	23·842	46	0·823	23·980
9	1·719	22·868	28	0·799	23·788	47	1·822	23·833
10	0·988	22·764	29	1·805	23·226	48	0·126	23·762
11	0·000	23·820	30	1·311	23·180	49	0·350	24·042
12	0·829	23·157	31	0·159	22·625	50	1·422	23·448
13	0·667	22·171	32	0·000	23·281	51	0·549	11·439
14	0·756	23·728	33	0·564	24·067	52	0·517	10·836
15	0·428	24·036	34	0·250	23·816	53	1·387	10·960
16	0·875	22·324	35	0·981	24·114	54	1·989	7·239
17	1·096	24·017	36	2·290	23·395	55	1·771	20·630
18	1·318	23·930	37	1·120	23·351	56	21·657	16·528
19	1·179	23·564	38	0·671	23·025		38·725	361·792
	24·621	423·230		15·530	445·686			38·725
		24·621			15·530			323·067
		398·609			430·156			430·156
								398·609

Height in feet, 1151·832

Height of waste-wear of Compensation Pond by barometrical measurement, ... 733·774

Height above the level of the sea in feet, 1885·606

The Black Hill to the west of Carnethy is about 20 feet higher, and as it appears to be the highest peak of the range, the highest elevation of the Pentland Chain is therefore 1900 feet above the level of the sea.

3. Let the heights on the poles taken by looking down the eminence be 11, 8, 5, 6, 4, and those taken by looking up be 5, 3, 1, 4, 6 feet. Required the height of the eminence.

Ans. 15 feet high.

* The surveyor should only have one series of back and fore observations on each page of his field-book, reserving a broad column on the right hand for remarks.

4. Let the heights taken by looking down be 10, 11, 7, 5, 8, 4, 9, and those taken by looking up be 3, 5, 2, 6, 4, 5½, 3½ feet. Required the height of the eminence. And, supposing the sloping distance from the bottom to the top to be 346 feet,—Required the height in a regular slope at the distance of 136 feet from the bottom.

Ans. 25 feet high in all, and, at 136 feet, 9·8266 feet.

TO MEASURE HEIGHTS BY THE BAROMETER.

The elasticity or the density of the air is as the weight of the superincumbent atmosphere; and therefore, if the heights vary in arithmetical progression, the densities will vary in geometrical progression; that is, the height is as the logarithm of the density. It has been found by experiment, that the module of the barometrical logarithms is 10,000 times that of the common logarithms; wherefore, if B be the height of the mercury at the lower station, and b that at the higher, and h the difference of the heights of the stations, then $h = 10{,}000 \times (\text{com. log. B} - \text{com. log. } b)$ expressed in fathoms. But this formula is true only upon the supposition that the temperature of the air is 32°, and that it is the same at both stations; neither of which is exactly true.

It is found by experiment, that quicksilver expands about $\frac{1}{10000}$ part of its bulk for every degree of Fahrenheit's thermometer. Let r be the temperature at the lower station, and r' that at the higher, as indicated by the thermometer attached to the barometer, then $b + \frac{r-r'}{10000}b$ will be the height of the mercury at the higher station, when reduced to the same temperature with that at the lower station; and thus

$$h = 10000 \times \left(\log. \mathrm{B} - \log. \left(b + \frac{r-r'}{10000}b\right)\right).$$

Again, the air expands nearly ·00223* of its bulk for every degree of Fahrenheit's thermometer. Let t be the temperature of the air at the lower station, and t' that at the higher, as indicated by a thermometer in the open air, then $\frac{1}{2}(t+t')$ may be taken for the mean temperature; and therefore the former formula has to be multiplied by $\cdot 00223 \times \left(\frac{t+t'}{2} - 32\right)$ for an additional correction.

PROB. XVII. To find the height of one place above another.

* General Roy's experiments gave ·00244, and Laplace's ·00232: the mean of the whole is ·00223.

From what has been shown, the complete formula will be $h = 10000 \times \left(\log. B - \log. \left(b + \frac{r - r'}{10000} b\right)\right) \times \left(1 + \cdot 00223 \times \left(\frac{t + t'}{2} - 32\right)\right)$, which, expressed in words, gives the following

Rule. Divide the difference of the heights of the attached thermometer by 10000, and add 1 to the quotient, and add the logarithm of the sum to the logarithm of the height of the barometer at the highest station, and subtract the sum from the logarithm of the height of the barometer at the lower station: the remainder, multiplied by 10000, will give the approximate height. Take the difference between 32° and half the sum of the heights of the detached thermometer, and multiply it by ·00223; and if the half sum of the heights be greater than 32°, add the product to 1, otherwise subtract; and the sum or remainder, multiplied by the approximate height, will give the true height very nearly.

Note. This method of finding heights is more convenient, but it is not so accurate as that of levelling.

1. Suppose the height of the mercury in the barometer at the bottom of the hill to be 29·56 inches, and at the top 28·27 inches, and the temperature of the mercury 63° and 54°, and the temperature of the air 56° and 48°. Required the height of the hill.

Ans. $\frac{63 - 54}{10000} = \cdot 0009$ and $10000 \times (\log. 29{\cdot}56 - \log. 28{\cdot}27 - \log. 1{\cdot}0009) = 10000 \times (1{\cdot}4707044 - 1{\cdot}4513258 - 0{\cdot}0003907) = 10000 \times \cdot 0189879 = 189{\cdot}879$ fathoms $= 1139{\cdot}274$ feet, the approximate height. Also, $\frac{1}{2}(56 + 48) - 32 = 20$, and $1 + 20 \times \cdot 00223 = 1{\cdot}0446$; therefore $1139{\cdot}274 \times 1{\cdot}0489 = 1190{\cdot}0856$ feet, the true height.

2. Let the height of the barometer at the lower station be 29·57, and at the higher 28·7 inches, the height of the attached thermometer at the lower 55·28°, and at the higher 51·75°, and the temperature of the air at the lower 54°, and at the higher 50·5°. Required the elevation. Ans. 803·684 feet.

3. Let the heights of the barometer be 29·4 and 25·19 inches, the attached thermometer 50° and 46°, and the temperature of the air 45° and 39°. Required the elevation.

Ans. 684·3787 fathoms.

4. Let the heights of the barometer be 29·89 and 26·27 inches, the attached thermometer 56·5° and 42·75°, and the

temperature of the air 55·25° and 43°. Required the ele tion. Ans. 3455·2375 f

PROB. XVIII. To measure distances by sound.

RULE. Multiply the time the sound takes in second 1142 : the product will be the distance in feet.

NOTE. Sound in common air moves uniformly at the rate of a 1142 feet in a second. Cold, and uneven surfaces, retard its m a little, and heat accelerates it in a small degree.*

1. I observed the flash of a gun 30 seconds before I h the report. How far was it distant from me?

Ans. $30 \times 1142 = 34260$

2. I observed a flash of lightning, and after 6 strok my pulse I heard the thunder, and my pulse makes 68 st in a minute. How far was the thunder distant from me

Ans. 1 mile 255·3 y

3. How long, after firing a gun, will it be till the rep heard at the distance of 8 miles? Ans. 37 sec

4. A person standing on the bank of a river heard the of his voice reflected from a rock on the opposite bank seconds after he uttered it. What was the breadth o river? Ans. 2284

PROB. XIX. To measure a height by the descent stone, &c.

RULE. Multiply the square of the time of descent conds by $16\frac{1}{12}$: the product will be the height in feet.

To find the time of descending. Divide the height i by $16\frac{1}{12}$, and the square-root of the quotient will be the in seconds.†

1. A stone takes 3 seconds in falling from the top of a to the ground. What is the height of the tower?

Ans. $3 \times 3 \times 16\frac{1}{12} = 144\frac{3}{4}$

2. In what time will a stone dropt from the height o feet reach the ground? Ans. 6 se

* A commission of the French Academy of Sciences in 1822 found b riment that the velocity of sound, when the temperature of the air Fahrenheit, was 1118 feet per second, and that every increase or decr temperature of 1° of Fahrenheit caused an increase or decrease of vel $1\frac{14}{100}$ foot per second.

† It has been found by accurate experiments, that a heavy body in t tude of London descends $16\frac{1}{12}$ feet in the first second of time, and the descended by falling bodies are as the squares of the times.

3. What is the height of a precipice, when a stone takes 7 seconds in falling from the top to the bottom?

Ans. $788\frac{1}{12}$ feet.

4. I reckoned 7 strokes of my pulse during the falling of a stone from the top of a rock. What height did it fall, the pulse beating 70 times in a minute. Ans. 579 feet.

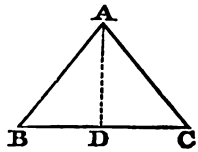
5. While a stone descended from the top of a tower, a pendulum 10 inches long made 8 vibrations. Required the height.*

Ans. 262·995 feet.

TO SURVEY FIELDS.

Prob. XX. To survey a triangular field ABC.

With the Chain. Measure the three sides by Prob. I.



With the Chain and Cross. Measure along BC by Prob. I., and with the cross find the point D, where the perpendicular from A meets BC, by Prob. VI. Write down the measures of BD, BC, and DA.

With the Theodolite and Chain. Measure one angle ABC by Prob. III., and the containing sides AB and BC by Prob. I. Or measure BC by Prob. I., and two angles ABC and ACB by Prob. III. From these measures the plan may be easily drawn by Prob. XIX. XX. or XXI. of Practical Geometry; and the area may be found by Prob. IV. V. or VI. of Mensuration of Surfaces.

1. In a triangular field I measured the base 856 links, and found the extremity to be the foot of the perpendicular upon it, which I measured 672 links. Required the content.

Ans. 287616 sq. links = 2 ac. 3 ro. 20 per. 5 yds. 5·53 sq. ft.

2. In measuring the base of a triangular field, I found the foot of the perpendicular 256 links from its extremity, the base 927, and the perpendicular 582 links. Required the area.

Ans. 269757 sq. links = 2 ac. 2 ro. 31 per. 18 yds. 4·4 ft.

3. I measured an angle of a triangular field 73° 24′, and the sides containing it 688 and 492 links. Required the area.

Ans. 162119·4 sq. links = 1 ac. 2 ro. 19 per. 15 yds. 4 ft.

4. I measured one side of a triangular field 1268 links, and took the angles at its extremities 57° 36′ and 62° 24′. Required the area.

Ans. 694579·4 sq. links = 6 ac. 3 ro. 31 per. 9·893 yds.

Note. Add the log. of the side and of its half to the log. sin. of the two angles, and the arithmetical complement of the log. sin. of

* The number of vibrations made by pendulums in the same time is as the square roots of their lengths.

the third angle; the number answering to the sum is the area required.

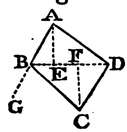
5. The three sides of a triangular field are 1275, 987, and 642 links. Required the area.

Ans. 311128 sq. links = 3 ac. 17 per. 24 yds. 3·1068 ft.

PROB. XXI. To survey a field contained by four sides.

WITH THE CHAIN. Measure the four sides and a diagonal BD by Prob. I.

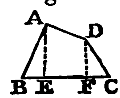
Fig. 1.



WITH THE CHAIN AND CROSS. Measure along a diagonal BD by Prob. I., and, with the cross, find by Prob. VI. the points E and F, upon which the perpendiculars fall from A and C, and write down the lengths of BE, BF, BD, then measure AE, and CF.

Or measure the longest side BC, marking E and F the places of the perpendiculars, and measure AE and DF.

Fig. 2.



WITH THE THEODOLITE AND THE CHAIN. Place the theodolite at B (fig. 1.) and take the angles ABD and DBC by Prob. III., and measure the diagonal BD by Prob. I., and again at D take the angles ADB and BDC. Or take the angle ABC, and measure the four sides.

If the angle ABC cannot be measured conveniently within the field, fix a pole G in the direction of either side AB, extended beyond B, and measure the angle CBG, which, subtracted from 180°, will give ABC.

WITH THE PLANE-TABLE AND THE CHAIN. Place the table at one of the angles B, from which all the other angles may be seen, and turn it round till the needle points to the fleur-de-lis, and there fix it. Fix also a pin in some part of the paper to represent B. Apply the fiducial side of the index to the pin, and turn it till the angle A is seen through the sights. Draw a line from the pin in that direction. Measure BA, and by the scale on the index lay it on that line from B to A. Next turn the index till the angle D is seen through the sights, and draw a line in that direction, and on it lay the length of BD. Then draw a line in the direction of C, and on it lay BC, and join CD and DA. In the same manner any field may be surveyed by the plane-table, when an angle can be taken, from which all the other angles of the field are seen.

1. I measured along the diagonal BD (fig. 1.), and at E 118 links from B, was the foot of the perpendicular AE 318,

and at F, 527 links from B, was the foot of the perpendicular CF on the opposite side of BD, 426 links: the whole length of the diagonal BD was 968 links. Required the plan and the area.

Ans. Area, 360096 sq. links = 3 ac. 2 ro. 16 per. 4 yds. 5·8176 feet.

2. I measured along BC the longest side of a four-sided field ABCD (fig. 2.), and at E, 125 links from B, was the foot of the perpendicular AE, which measured 624 links, and at F, 635 from B, was the foot of another perpendicular FD 462 links: the whole length of the side BC was 1274 links. Required the plan and the area.

Ans. Area, 463539 sq. lin. = 4 ac. 2 ro. 21 per. 20·0376 yds.

3. I measured an angle ABC of a quadrilateral field 128°, and the four sides AB 536 links, BC 843, CD 634, and AD 936 links. Required the plan and the area.

Ans. Area, 466592·7 sq. links = 4 ac. 2 ro. 26 per. 16 yds. 5·28 feet.

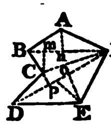
4. I measured the diagonal BD of a four-sided field 1462 links, and at its extremities I took the angles which it made with the sides, viz. ABD 48° 20′, CBD 41° 26′, ADB 29° 40′, and BDC 38° 44′. Required the plan and the area.

Ans. Area, 853086 sq. links = 8 ac. 2 ro. 4 per. 28 yds. 8·2616 feet.

5. In taking the plan of a quadrilateral field by the Plane-Table, I found the straight side AB to lie N. 73° E., and to measure 568 links; the diagonal AC to lie S. 83° E., 978 links; and the side AD to lie S 47° E., 734 links. Required the plan and the area.

Ans. Area, 323942·9 square links = 3 ac. 38 per. 9 yds. 3·02724 feet.

PROB. XXII. To survey any field with the chain.



Measure all the sides of the field, and then the diagonals BF, FC, FD. From these the field may be drawn upon paper by Prob. XXVIII. of PRACTICAL GEOMETRY, and its area may be found by Prob. XI. of MENSURATION OF SUPERFICIES.

Or divide the field by diagonals into as many trapezes as possible, and the remainder will consist of one or more triangles. Thus the field ABCDEF may be divided into two trapezes ABCF and CDEF, by joining CF. These may be surveyed as in the last Problem.

1. In a six-sided field I measured all the sides, viz. AB 583 links, BC 324, CD 456, DE 892, EF 728, and AF 477

links, and from F measured the diagonals FB 897, FC 723, and FD 948 links. Required the plan and the area.

Ans. Area, 700266·04 sq. links = 7 ac. 12·876 yds.

2. In a heptagonal field I measured along the northernmost diagonal BG, and at 207 links from B found the foot of a perpendicular above it AH, which measured 272; and at 578 from B found the foot of a perpendicular under it FK, which measured 498; the diagonal BG 928. From F, I measured along a diagonal FC, and at 488 from F was the foot of the perpendicular from B, which measured 587, and the diagonal FC 896. Then, from C, I measured along a diagonal CE, and at 498 from C was the foot of an under perpendicular ND 630, and at 688 from C was the foot of a perpendicular FM 574 links; the diagonal CE was 1093 links. Required the plan and the area.

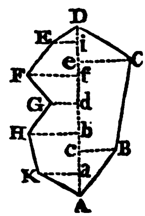
Ans. Area, 1278242 sq. links = 12 ac. 3 ro. 5 per. 5 yds. 5·9652 feet.

NOTE. 1. If a perpendicular, as E*p*, upon a diagonal DF, fall without the field, and it be inconvenient to measure it in that situation, the other diagonal CE, with the perpendiculars upon it, may be taken; or the two triangles DEF, CDF, may be measured separately.

3. In a hexagonal field ABCDEF, I measured along the diagonal BF, and, at 328 links from B, I was at the foot of the perpendicular AG, which measured 286, and the diagonal BF was 536; but had to measure 127 links farther without the field, to come to the foot of the perpendicular EH on the opposite side of BF, which measured 453. Again, measuring along the diagonal EC, I found, at 386 from E, the foot of the perpendicular DK, which measured 496; and, 674 from E, found the foot of the perpendicular BL, which measured 436; the whole length of the diagonal EC was 895 links. Required the plan and the area.

Ans. 615122 sq. links = 6 ac. 24 per. 5 yds. 8·1432 ft.

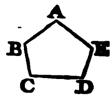
NOTE 2. In fields not very large it will be sufficient to measure one diagonal, and the perpendiculars upon it from all the other angles.



4. Suppose the distances of the perpendiculars from A to be 50, 145, 220, 295, 380, 475, and 655, the whole line AD being 725 links, the second and sixth distances reach to perpendiculars on the right hand, and the rest to those on the left hand. Also the perpendiculars on the right are 75 and 150, and the others in their order are 110, 135, 85, 275, and 185 links. Required the plan and the area.

Erect perpendiculars upon AD, at their proper distances from A; and, having made them of their proper length, the plan is drawn by joining their extremities. The area is found by Prob. IV. and VII. of MENSURATION OF SURFACES to be 178162·5 sq. links = 1 ac. 3 ro. 5 per. 1 yd. 7·335 ft.

PROB. XXIII. To take the plan of a field by going round it.



WITH THE PLANE-TABLE. Place the table at a corner A, and fix it when the needle points to the fleur-de-lis, and take a point A on the paper. Direct the index from the assumed point to the corner E of the field, and draw a line; then direct the index to B, and draw another line. Measure the lines in the field from A to B and from A to E, and lay these lines on the paper. Place the table at B, and, laying the index along BA on the paper, turn the table about till A is seen through the sights; the needle ought then to point to the fleur-de-lis. Direct the index to the corner C of the field, and draw a line, on which lay the length of BC. In the same manner are to be laid down the position and the lengths of the other sides CB and DE, and the last line will terminate at E on the paper, if no error has been committed.

WITH THE THEODOLITE. Place the instrument at the corner A of the field, and, having turned it till the needle points to the fleur-de-lis, take the bearing of one of the sides, as AE; then observe the angle EAB, and measure AB. Again, place the theodolite at the corner B, and observe the angle ABC, and measure BC. Proceed in this way to take all the angles and to measure the sides.

NOTE 1. Add all the angles together, if they be interior; but if any of them be exterior, add the difference between it and 360°: the sum should be equal to 180°, multiplied by the number of sides, wanting two. The error, if any, should be equally divided amongst the angles.

NOTE 2. If the interior angles cannot be taken, let the exterior be taken by extending the direction of the sides. The sum of all the exterior angles should be 360°; but if any of the corners point inward, add 180° to 360° for every such angle, and the sum should be the sum of the angles.

NOTE 3. The things measured for laying down the plan of a field will always be sufficient for finding its content, but they will not always afford the shortest method. Thus, in taking the plan of the pentagonal field ABCDE by measuring the sides and angles, if we draw diagonals AC and CE, we can find the area of the triangle ABC from the sides AB and BC and the angle B, that of the triangle CDE from the sides CD and DE and the angle D; but then we have nothing given in the triangle ACE from which to find its area. We must therefore find, by trigonometry, in the triangle ABC, the angle ACB and the base AC, and in the triangle CDE, the angle DCE

and the base CE; and these two angles, subtracted from BCD, will give the angle ACE, from which, with the sides AC and CE, we can find the area of the triangle ACE. And thus, by the help of trigonometry, we may find in every case sufficient data for computing the area from the things measured for taking the plan.

1. Let AB be 750, BC 810, CD 628, DE 598 links, and the angles at B 72°, at C 136°, and at D 122°. Required the area.

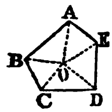
The angles are ACB 50° 58′ 11″, DCE 28° 13′ 23″, and ACE 56° 48′ 26″, and the sides AC 918·23, and CE 1072·38 links.

Ans. Area, 860133 sq. links = 8 ac. 2 ro. 16 per. 6 yds. 3·9348 ft.

2. In a six-sided field ABCDEF, let AB be 482, BC 586, CD 760, DE 812, and EF 910 links, and the angles at B 96°, at C 132°, at D 146°, and at E 106°. Required the area.

Ans. Area, 1500073·62 sq. links = 15 ac. 3 yds. 5·07 ft.

PROB. XXIV. To survey a field from a station within it.



The station must be chosen such that all the angles may be seen from it.

WITH THE PLANE-TABLE. Place the table at O, from which all the corners may be seen, then turn it to bring the needle to the fleur-de-lis; and on the paper take a point O, to represent the station. Direct the index from O to the corner A, and draw a straight line to represent OA in the field. Draw, in the same manner, lines to represent OB, OC, &c. Then measure from the station to A, B, C, &c. in the field, and lay them on their representatives, and join their extremities.

WITH THE THEODOLITE. Place the instrument at the station O, and, putting the needle to the fleur-de-lis, take the bearing of OA. Next observe the angles AOB, BOC, &c. the sum of which should amount to 360°. Then measure straight lines from O to A, B, C, &c.

1. Suppose OA 798, OB 459, OC 434, OD 852, and OE 912 links, and the angles at O, AOB 74°, BOC 38°, COD 102°, DOE 82°, and EOA 64°. Required the area.

Ans. 1130004·3 sq. links = 11 ac. 1 ro. 8 per.

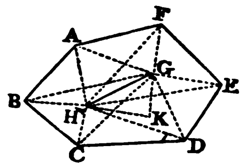
2. In a heptagonal field I found the angles at the instrument to be 67°, 43°, 84°, 56°, 27°, 51°, and 32°, and the distances of the angles from the instrument to be 528, 632, 916, 478, 732, 830, and 816 links. Required the plan and area.

Ans. Area, 1228999 square links = 12 ac. 1 ro. 6 per. 12·0516 yds.

PROB. XXV. To survey a field from two stations.

The stations must be such that all the objects to be laid down on the plan may be seen from them both, and that the angles which they make with the line joining the stations may not be too small.

The stations may be taken either within the boundaries of the field, in one of the sides, in the direction of two of the objects to be laid down, or at a distance, and without the boundaries of the field to be surveyed.



WITH THE PLANE-TABLE. Place the table at one of the stations, and the needle to the fleur-de-lis, then take a point G on the paper to represent that station, and direct the sights of the index from it to the other station; draw GH, and on it lay the distance between the stations from G to H. Direct the sights from G to the corner A; draw GA with a black-lead pencil, and upon any part of it place the letter A. Again direct the sights from G to the corner B; draw GB, and on it write B. In the same manner draw GC, GD, &c.

Remove the table to the second station, and turn it till the needle points to the fleur-de-lis; then the index, laid on HG of the paper, will point to the former station. Direct now the sights from H to the corner A; draw HA, which will meet the line GA in the point representing that corner, at which place A, and erase the former A. In the same manner draw HB, meeting GB in B, and so on; then join AB, BC, &c. In the same way the position of any other thing, as the house K, may be determined by drawing GK towards it when the table is at G, and HK towards it when the table is at H.

WITH THE THEODOLITE. Place the instrument at the first station G, and turn it till the needle points to the fleur-de-lis; take the bearing of the station H, and measure GH. Then take the angles HGC, CGD, DGE, &c., and lastly BGH. Remove the instrument to the second station H, and bring the needle to the fleur-de-lis; then the station G ought to bear upon the point opposite to that upon which H bore from G. If it does, take first the angle GHF, then FHA, AHB, &c., and lastly EHG. The sum of the angles taken at each station ought to be exactly 360°.

Every thing else which is to be put in the plan must be surveyed in the same way, by taking at G the angle between GH and the line from G to it, and the same at H. All these observations must be recorded in the field-book.

When the whole cannot be seen from two stations, more sta-

tions must be chosen. The lines between the stations must be measured, and the angles observed as before. But care must be taken to determine the position of each of the lines joining the stations.

In this manner, not only may fields be surveyed without even entering them, but a map may be made of the principal parts of an estate, or even of a county, and the chief places of a town, or any part of a river or coast, may likewise be surveyed by taking two such stations.

1. Required the plan and the area of a field from the following

FIELD-BOOK.

Angles at G.	Angles at H.	Remarks.
C 22° 0′	F 20° 0′	GH bears S. 67° 30′ W.
D 86 30	A 72 0	1038 links.
E 146 30	B 145 0	Corner of a house at K.
F 232 30	C 243 0	Angles at G 50°
A 313 30	D 317 0	at H 323°.
B 348 30	E 344 0	
H 360 0	G 360 0	

In this field-book, the angles at G are marked as taken with the theodolite when placed at that station. The sights, when at the beginning of the degrees, were directed to the station H, and the instrument fixed there. Then the moveable index was turned to C, and cut off 22° for the angle HGC, which, in the field-book, is marked C, the other two letters being found at the top; then it was turned to D, and cut off 86° 30′ for the angle HGD; and the difference of these two is the angle CGD. It was then turned to E, and cut off 146° 30′ for the angle HGE; and so on all the way round. In the same way the angles were taken at H, both for determining the corners of the field and for finding the corner of the house at K.

In calculating the areas of fields surveyed from more than one station, it is necessary to calculate, by trigonometry, the length of all the lines drawn from one of the stations to the angles; and for this purpose we have, in every triangle of which GH is a side, all the angles and this side to find the other side; after which the area is found as in the preceding problem. Here the distances from G are GA 1123·3, GB 1493·1, GC 1409·73, GD 917·43, GE 951·44, and GF 660·74

links; from which the areas of the triangles AGB, BGC, CGD, DGE, EGF, and FGA, are to be calculated.

Ans. 2703648·8 sq. links = 27 ac. 5 per. 25 yds. 3·167 ft.

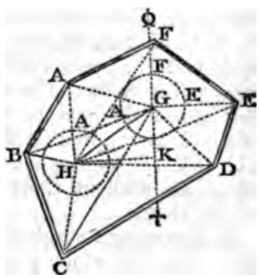
2. Required the plan and the area of a field from the following

FIELD-BOOK.

Angles at P.	Angles at R.	Remarks.
F 3°	A 6°	PR bears S. 22° 30′ E. 1827 links.
E 28	H 24	
D 49	G 64	
C 65	F 186	
B 132	E 228	
A 197	D 271	
H 247	C 319	
G 320	B 342	
R 360	P 360	

Ans. 10037324·8 sq. links = Area, 100 ac. 1 ro. 19 per. 21 yds. 3·08128 ft.

PROB. XXVI. To draw the plan of the field upon paper from the field-book.



Draw a faint line up and down the paper to represent the meridian, the upper end the north, and the under end the south. Using the data given in Ex. 1, Prob. XXV., in this line take a convenient point G for the first station. On the south side of G make an angle of 67° 30′ towards the left hand, which will give the position of GH; and take 1038 from any convenient scale, and lay that extent from G to H, to get the station H. The best protractor for laying down the angles is a circular one, divided into 360°. Place the centre at G, and the beginning of the degrees on GH. Make a mark at 22°, and at it write a faint C; make another mark at 86° 30′, and there write a faint D, and so on all the way round; and draw faint lines from G through the marks. Next place the centre of the protractor at H, and the beginning of the degrees on GH; and at 20° make a mark, and

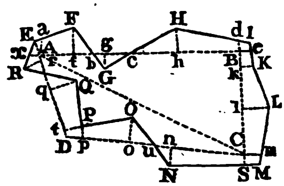
write F; at 72° make a mark, and write A, and so on; and draw lines from H through the marks. The lines from G and H, through the points where the same letter is written, must be drawn out till they meet, and their intersection is at the angle to which that letter belongs. Thus GA and HA will meet in the angle A, GB and HB will meet in the angle B, &c. After this join AB, BC, &c. for the boundaries of the field.

If the protractor be a semicircle, then, after laying down the angles less than 180°, the protractor must be laid on the other side of GH, and 180° taken from each of the remaining angles before they are laid down.

Prob. XXVII. To survey fields with crooked boundaries.

The boundaries of fields are seldom straight lines, and therefore surveyors generally erect poles near the corners of the ground to be surveyed, and conceive these poles joined by straight lines. This constitutes the body of the field; and the parts between these lines and the boundaries are considered as offsets, and their areas found separately.

The points, therefore, which, in the preceding problems, were called angles or corners, are to be considered only as the places of these poles, and the fields surveyed as contained by the lines joining them; and to complete the survey, the situation and distance of the boundaries from these lines must be found.



1. Let EIMP be a field to be surveyed. Poles are erected at A, B, C, D, near corners of the field, and the space ABCD is surveyed as before. The rest of the field is obtained by taking offsets from the lines AB, BC, CD, DA, and adding the spaces which are without these lines, and taking away the spaces within them.

In surveying a single field, an outline of it may be sketched upon paper, on which the dimensions may be written down as they are found. But in surveys of large estates, counties, &c. a field-book must be used, for registering all observations and dimensions. The field-book generally consists of three columns: the middle one contains the distance measured along the main lines AB, BC, &c.; and the other two are for the offsets, according as they are on the right or left of the main line. For this purpose it is best to begin at the bottom of the field-book, and to write upwards, that the offsets on the right side of the main line may be placed in the right-hand column,

FIELD-BOOK.

Left offsets.	Main lines.	Right offsets.
AC, S. 60° 25′ E. 1896.		
	844	Including offset to cor.
86	746	Close to A.
152	688	
	594	
	462	200
D	64	90
	1410	D Γ
	1362	92
	924	196
	744	
146	600	
C 48	0	
> 108		C Γ
104	912	
264	508	
84	152	
B 70	0	
> 128		B Γ
94	1672	
172	1166	
	752	
	530	108
	442	
200	284	
A 106	0	
To left.		To right.

ıe offsets on the left ı the left-hand co- Thus, in measuring A to B, the offset which measures 106 is on the left hand B, at the beginning line; therefore write the middle column, bottom, and oppo- it, in the left-hand n, write 106. Then ıring along AB, the f is to be found, upon the perpendicular rom F: this is 284 from A, and *f* F is nks; therefore write in the middle co- , and 200 opposite to he left-hand column. ı, at 442 links from he line AB crosses boundary-line FG; fore write 442 in the le column, and in the ent columns draw ght lines in the di- on of the straight line ıearly, for the exact ion of it is not re- d at this stage of the ey. At 530 the per- icular from G meets and G*g* is 108; place efore 530 in the middle column, and 108 opposite to it in ight-hand column.

roceed in this way to B, where, besides the offset, BI is sured, and placed in the left-hand column, with the mark o show that it is not perpendicular. At the same place ıe right-hand column is placed the mark Γ, to show that the surveyor turns to the right hand. This finishes the ey along the line AB, and a line is drawn across the book parate it from the next line. Proceed in the same way ı B to C, from C to D, and from D to A.

The position of any one of the lines, as AC, being fi[xed] with the compass, it will determine the position of the wh[ole]. But in using the compass, the variation should be allow[ed], and great care ought to be taken lest the needle be attract[ed] by some metallic substance in its neighbourhood.

Ans. 1462335·12 sq. links = 14 ac. 2 ro. 19 per. 22·27 yd

(2.) FIELD-BOOK.

Left offsets.	Main lines.	Right offsets.
Diagonal AC, bears N. 28° W. 760 links.		
0	660	
30	450	
D 0	400	
0	490	D Γ
10	400	
40	300	
55	200	
C 20	50	
	635	0 C Γ
	500	25
	400	30
	300	
50	200	
B 40	100	
0	395	B Γ
20	350	
35	300	
45	250	
50	200	
30	100	
A 15	50	

Ans. 3·1764515 acres.

(3.) FIELD-BOOK.

Left offsets.	Main lines.	Right offsets.
Diagonal AC, bears S. 56° E. 1560 links.		
	1350	
0	1200	
40	900	
20	750	
60	550	
85	400	
70	350	
D 35	200	
0	800	D Γ
34	700	
	500	
	350	80
C	200	60
B	1100	C Γ
0	912	B Γ
40	800	
	750	
	680	50
	600	
90	450	
A 50	340	

Ans. 10·816122 acres.

Lay down the plans of the following properties from the field-book for the three examples, and calculate their contents.

Fig. 1.

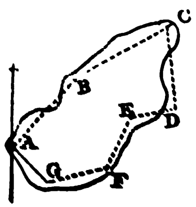
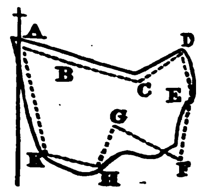


Fig. 2.



(4.) (Fig. 1.)

Diagonals.
BD 1100
BE 720
BF 1080
AF 1000
AB bears N. $37\frac{1}{4}°$ E.

20	510	A ┌
200	360	
20	0	
20	612	G ┌
156	320	
30	0	
30	600	F ┌
70	256	
20	0	
20	480	E ┐
28	220	
	114	
	0	30
D ┌	920	36
	826	78
	560	340
	356	90
	281	
120	180	
30	0	
25	900	C ┌
40	728	
120	560	
57	256	
20	0	
20	1040	B ┌
56	980	
	826	
	673	56
	522	
210	443	
120	156	
20	0	A

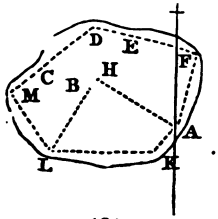
s. 1503446·3 sq. links = 15
. 5 per. 15 yds. 4·95828 ft.

(5.) (Fig. 2.)

Diagonals.
CE 620 | GB 850
CF 1000 | GK 710
CG 610 | BK 940
AK bears S. 11° E.

20	1150	A ┌
25	680	
35	420	
50	0	
60	580	K ┌
90	500	
150	300	
100	0	
89	470	H ┌
130	260	
200	0	
400	800	G ┐
380	630	
220	480	
36	230	
	153	
	110	25
	0	40
F ┌	760	50
	640	78
	520	115
	380	85
	200	40
	86	
30	0	
30	420	E ┌
35	320	
30	100	
20	0	
25	500	D ┌
89	360	
72	150	
30	0	
40	730	C ┐
150	540	
110	210	
30	0	
20	450	B ┐
70	250	
30	0	A

Ans. 1839891·2 sq. links = 18 ac. 1 ro. 23 per. 24·984 yds.



(6.)

	2180	A	
15	626	15	S. 59° E.
	426	H	
20	0	10	
20	1610	10 B Γ	
20	1590		N. 29° E.
	0	L	
To houses.			
A Γ	2050	15	
	1969		S. 13° W.
180	1000		
9	0		
51	1380	F Γ	
120	600		S. 77° E.
20	0		
20	750	E Γ	
24	500		S. 85° E.
10	0		
10	1400	D Γ	
500	1000		
400	700		N. 51° E.
300	400		
	25		
	0	20	
15	655	C Γ	
10	0		N. 45° E.
10	1450	M Γ	
350	600		N. 31° W.
20	0		
20	2280	L Γ	
220	1400		N. 85° W.
10	0		
10	640	K Γ	
100	400		N. 36° W.
20	0	A	

Ans. 89·259682 acres.

Prob. XXVIII. To take an extensive survey.

Choose for stations the most eminent places, from which the principal parts of the survey may be seen. Particularly choose such eminences as lie near the boundaries. Take the angles which these stations make with one another with great accuracy, and measure carefully in a straight line the distances from station to station, marking the places where the lines pass ditches, roads, rivulets, &c., and take offsets to near objects, leaving in the ground a mark at every place where you marked the distance in the field-book, distinguishing these marks by letters or figures, that they may not be mistaken for one another. In this way you will obtain the situation of the principal parts. Then take other stations within these, and measure the distances as before. And thus divide and subdivide the survey, till you come to single fields, which may be measured by some of the preceding methods.

The longer the distance is between the stations, if accurately measured, the more correct will the work be; but this cannot be ascertained by a single measurement, without using various methods of determining it. At the same time, an error in these primary distances affects the whole survey; and therefore every care ought to be taken to prevent it.

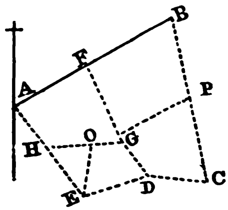
After the principal parts of the survey are laid down accurately, so as to have the whole divided into small compartments, these may be filled up by the plane-table, one by one.

In laying down the plan, proceed in the same way, first laying down the principal distances and the boundaries, and then the interior parts as they are surveyed; and, in filling up the particular departments, care must be taken to lay down the boundaries of parishes, estates, farms, &c. and to point out the particular situations of towns, villages, churches, gentlemen's seats, towers, farm-steads, also rivers, lakes, ponds, woods, plantations, rocks, precipices, and all the eminences, mines, pits, quarries, and in general every thing which can contribute to give a proper understanding of the nature of the survey. All these must be neatly sketched and properly coloured, and the names of the places printed in them.

1. I took two stations near a road, of which B lay from A, N. 61° E. 1850 links; and from A took the bearings of the eminences C, S. 70° E., D, S. 62° E., and E, S. 36° E., and at B took their bearings C, S. 14° E., D, S. 6½° W., and E, S. 26° W. Required their distances from the stations, and their bearings and distances from one another.

Ans. BC 1684·139, AE 1201·789, CD 596·638, and DE 753·3655 links.

Having drawn the plan of the observations in Example 1, it is required to lay down on it, and to calculate the properties contained in the field-book of the following examples



(2.)

Diagonal. FH 935		
35	560	A
100	320	
88	180	
20	0	
20	695	H Γ
60	513	
	313	
O	300	4
	0	5
	870	G Γ
105	450	
50	0	
4	900	F Γ
98	734	
150	540	
122	330	
40	0	A
	At the road.	

Ans. 7·286775 acres.

(3.

Diagonal. PF 1065.		
G	945	5
	878	80
	805	P Γ
44	366	
10	0	
10	950	B Γ
28	825	
90	740	
60	580	
30	430	
30	400	
78	260	
20	0	F
	At the road.	

Ans. 7·3035896 acres.

(4.)

Diagonal. PD 945.		
	540	G
	360	58
	260	80
	0	20
70	597	D Γ
98	350	
	0	
	879	C Γ
203	621	
170	421	
	0	P

Ans. 6·503221 acres.

(5.)

Diagonal. EG 670		
4	564	O Γ
70	372	
130	248	
65	100	
12	0	
12	753	E Γ
90	613	
160	518	
170	416	
150	298	
40	0	D

Ans. 4·071447 acres.

The distances not mentioned in these two examples are to be taken from the preceding ones.

PROB. XXIX. To find the contents of a survey.

The areas of single fields, bounded by straight lines, may be found from the lines measured in the field, by the first twelve problems of MENSURATION OF SURFACES.

TO CALCULATE OFFSETS. The most accurate method is to compute them separately, as triangles and trapezoids, by Prob. IV. and VII. of MENSURATION OF SURFACES.

METHOD 2. If the distances between the perpendiculars be nearly equal. To half the sum of the perpendiculars at the extremities of the base add all the rest, and multiply the sum by the base, and divide the product by the number of divisions in the base made by these perpendiculars.

COMMON METHOD. Divide the sum of the perpendiculars by the number of them for a mean perpendicular, by which multiply the base.

If the boundary be a curve-line, and the distances between the perpendiculars equal, the area may be calculated by Rule II. Prob. XXV. of MENSURATION OF SURFACES.

The fourth Example in Prob. XXII. wrought by the first Method.

50×110	$=$	5500	for the triangle	AKa
$170 \times (110+135)$	$=$	41650	.... trapezoid	KabH
$75 \times (135+85)$	$=$	16500	.... trapezoid	HbdG
$85 \times (85+275)$	$=$	30600	.... trapezoid	GdfF
$275 \times (275+185)$	$=$	126500	.... trapezoid	FfiE
70×185	$=$	12950	.... triangle	EiD
145×75	$=$	10875	.... triangle	ABc
$330 \times (75+150)$	$=$	74250	.... trapezoid	BceC
250×150	$=$	37500	.... triangle	CeD
		2)356325		
	Ans.	178162·5	the whole area.	

By the second Method.

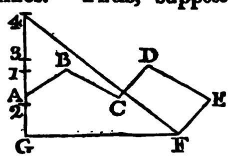
Ans. $725 \times (\frac{1}{8} \times (110 + 135 + 85 + 275 + 185) + (150 + 75) \times \frac{1}{3}) = 149833\frac{1}{3}$ area.

By the third Method.

Ans. $725 \times (\frac{1}{5} \times (110 + 135 + 85 + 275 + 185) + (150 + 75) \times \frac{1}{2}) = 196112{\cdot}5$ area.

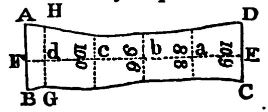
Some surveyors endeavour first to obtain a correct plan of the land, and then they measure, on the plan, such lines as will enable them to calculate its contents with the greatest expedition; and for this purpose they reduce the crooked boundaries to straight lines. Sometimes this is done by stretching a hair through the crooked part, so that the small parts cut off by the hair may be equal to the parts taken in, as nearly as the eye can judge; and though this can be done very nicely by an experienced surveyor, it should never be trusted to, when it is possible to have the whole measured in the field.

Others reduce the crooked parts to a triangle, by Prob. XXXIV. of Practical Geometry, which can be done by the parallel ruler without drawing lines. Thus, suppose ABCDEFG to be the space which is to be reduced to a triangle. Lay the parallel ruler from A to C, and move it till it pass through B, and mark the point 1 in which it cuts AG, or its extension. Lay the ruler through 1 and D, and move it till it pass through C, and mark 2 where it cuts AG. Again lay the ruler from 2 through E, and move it till it pass through D, and mark 3 where it cuts AG, and so on; then join 4 and F, and the triangle F4G is equal to the given space. For B1 is parallel to AC; therefore if C1 were drawn, the triangle AC1 = ACB. Now, when the ruler passes through A and C, it



takes in the triangle ACB; and when it is moved to B1, it cuts off the triangle AC1. In like manner the triangle 1D2, which is taken in, is equal to the triangle 1DC cut off; and so of the rest.

Another method of calculation practised by surveyors is the following, which, though it depend upon judgment, will be found to come very near the truth, and is very expeditious.



Let ABCD be the plan of a survey, and DC a straight boundary. Draw EF perpendicular to DC, and on it lay a chain, from E to *a*, from *a* to *b*, from *b* to *c*, &c.; and draw parallels to CD through *a*, *b*, *c*, &c., and they will divide the plan into spaces, each a chain in breadth. Measure in a line parallel to DC, half-way between E and *a*. This is supposed to give the mean length of the first space, and therefore is to be measured where the length is a mean, as nearly as the eye can judge. It is here supposed to be 109 links, and is written so in the first space. In the same manner the mean lengths are taken in all the other divisions. After this these lengths are to be added together, and require only three places to be cut off to give the area in acres. The small space ABGH remaining beyond the last parallel, which is only 39 links in breadth, may be found by multiplying 39 by its mean length, judged of as before. Or offsets upon GH may be taken from A and B, and thus a mean breadth may be obtained, to be multiplied by GH, or the mean length. Suppose the offsets at A and B to be 44 and 31, and suppose the mean length to be 96 links; then $96 \times 39 = \cdot 03744$ of an acre. Or the mean offset is 37·5, which, multiplied by GH, suppose 100, gives ·03750 of an acre for the content of the part ABGH; and this, added to ·393, the sum of the mean lengths of the other pieces, gives ·4305 of an acre, or 1 rood 28·88 perches, for the whole area.

Prob. XXX. To measure and plot hilly ground.

Rule. The surface is measured as in level ground; but we must lay down on the plan only the area of the base on which the hill stands. Now the length of the base or plotting-line is found by this proportion, radius : cos. of the angle of acclivity : : the surface-line : the base-line.

1. A line of 1200 links is measured up a hill whose angle of acclivity is 12° 15′, what is the length of the base-line?

Cos. 12° 15′ — rad.	= 9·989997
Surface-line, 1200 log.	= 3·079181
Base line, 1172·7 log.	= 3·069178

2. A line of 1764 links is measured up a hill whose angle of acclivity is 17° 20′, what is the length of the base-line?

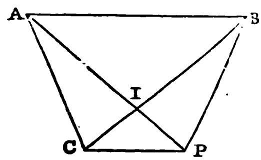
Ans. 1683·9 links.

3. The angle of acclivity of a hill is on the east side 25° 30′ and on the west side 20° 45′; a line from its base to the summit is on the east side 5000 links, and on the west side 5500 links, what is the length of the base-line?

Ans. 9656·1 links.

PROB. XXXI. To deduce from angles measured out of the station, but near it, the true angles at the station.

When the centre of the instrument cannot be placed in the vertical line occupied by the axis of a signal, the angles observed must undergo a reduction according to circumstances.



Let C be the centre of the station P, the place of the instrument, or the vertex of the observed angle APB, to find the angle ACB.

Supposing that APB = P, APC = p; CP = d; AC = L, and BC = R are known.

Since the exterior angle of a triangle is equal to the sum of the two interior and opposite angles; then the angle AIB = P + IBP, and AIB is also equal to C + CAP; hence P + IBP = C + CAP, and by transposition C — P = IBP — CAP; but the triangles CBP, CAP give

$$\text{Sin. CBP} = \text{sin. IBP} = \frac{\text{CP}}{\text{BC}}\,\text{sin. BPC} = \frac{d\,\text{sin.}\,(\text{P}+p)}{\text{R}}.$$

$\text{Sin. CAP} = \frac{\text{CP}}{\text{AC}}\,\text{sin. APC} = \frac{d\,\text{sin.}\,p}{\text{L}}$; and as the angles CBP, CAP are by hypothesis always very small, their sines may be substituted for their arcs; whence $\text{C} - \text{P} = \frac{d\,\text{sin.}\,(\text{P}+p)}{\text{R}} - \frac{d\,\text{sin.}\,p}{\text{L}}$.

When the reduction is required in seconds the equation becomes $\text{C} - \text{P} = \frac{d}{\text{sin.}\,1''} \times \left\{\frac{\text{sin.}\,(\text{P}+p)}{\text{R}} - \frac{\text{sin.}\,p}{\text{L}}\right\}$.

NOTE 1. In using this formula, the signs of p and of $(\text{P}+p)$ must be carefully attended to: thus the first term of the correction will be positive if the angle $(\text{P}+p)$ is between 0° and 180°, and negative if that angle exceed 180°; and the contrary will obtain in these circumstances with regard to the second term, which answers to the angle of direction p.

NOTE 2. When the signal is either a circular or polygonal tower, the method of obtaining the exact angle will suffer a slight variation, which will easily be understood by any one acquainted with the rudiments of Geometry.

PROB. XXXII. When a base-line is measured at an elevated level, to find its length when reduced to the level of the sea.

Let $r =$ the mean radius of the earth, or the distance from the surface to the sea-level, $h =$ the height above the level of the sea, at which the base is measured, $B =$ the measured base, and $b =$ that to which it must be reduced at the level of the sea; then since B and b are portions of similar and concentric circles to the radii $r+h$ and r, it is obvious that $r+h : r ::$ $B : b$ or $b = B \times \frac{r}{r+h}$; hence $B - b = B - \frac{rB}{r+h} =$ $\frac{Bh}{r+h} = B \times \left(\frac{h}{r} - \frac{h^2}{r^2} + \frac{h^3}{r^3} - \&c.\right)$; but the radius of the earth being extremely great in proportion to h, we may, for all practical purposes, assume $B - b = B \times \frac{h}{r}$.

RULE. Add the logarithm of the measured base in feet to the log. of its height above the sea also in feet, and the constant log. 2·680110, the sum will be the log. of the correction in feet, which is always subtractive.

NOTE. In order to arrive at the most accurate results in the practice of surveying, the following rules, which are demonstrated in the third volume of Hutton's Course, should be carefully attended to.

I. When only one side of a triangle is to be determined, the measured base should be as nearly equal to the side sought as possible.

II. When two sides of a triangle are to be determined, the triangle should be nearly equilateral.

III. When two sides are to be determined, and the base cannot be equal to either of the sides, it should be taken as long as possible, the two angles at the base should be equal, and not less than 23°.

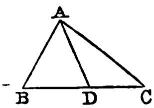
OF DIVIDING LAND.

PROB. XXXIII. To divide a triangular field ABC in any proportion, by a straight line drawn from the vertex A, to the opposite side BC.

RULE. Divide the base BC in the required proportion, by Prob. IX. PRACTICAL GEOMETRY, and draw a line from the vertex to the point found in the base.*

1. Divide the triangle BAC, of which the base BC is 950 links, in the ratio of 9 to 7, by a line drawn from the vertex A.

Ans. 16 : 7 : : 950 : 415$\frac{5}{8}$ to be laid from B to D; then AD is the dividing line.



2. Divide the triangle ABC, of which the sides are AB 386, BC 428, and AC 533 feet, in the ratio of 8 to 5, by a line drawn from the angle B. Ans. AD 328, and DC 205 feet.

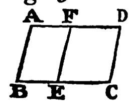
3. Divide the triangle ABC, of which AC is 374, and AB 473 links, and the angle BAC 54°, in the ratio of 5 to 6, by a line drawn from C. Ans. AD 215, and DB 258 links.

PROB. XXXIV. To cut off any portion from a parallelogram by a straight line parallel to one of the sides, having the other side given.

RULE. As the whole content is to the portion to be cut off, so is the length of the given side to the point through which the line of division must be drawn.

1. It is required to cut off 3 acres from a field ABCD of 10 acres, by a line parallel to AB, the side BC being 495 links.

Ans. 10 : 3 : : 495 : 148$\frac{1}{2}$ to be laid from B to E, and from A to F; then EF is the dividing line.



2. Divide the parallelogram ABCD, of which AB is 236, and BC 574 yards, and the angle ABC 76°, in the ratio of 3 to 4, by a line parallel to AB. Ans. BE 246, and EC 328 yards.

3. Divide the rectangle ABCD, of which AB is 472, and BC 675 feet, in the ratio of 7 to 8, by a line parallel to AB. Ans. BE 315, and EC 360 feet.

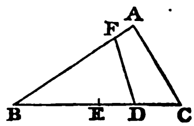
PROB. XXXV. To cut off any portion from a triangular field ABC, by a straight line drawn from any point D, in the side BC.

RULE. Find by Prob. XXXIII. the point E in the base, to which a line drawn from the vertex would divide the field in the given ratio. Then if E falls between B and D, the point

* It is manifest that the two triangles into which the field is divided have the same altitude; they are therefore as their bases (El. Geom. XVI. Cor.)

F will be in BA, otherwise it will be in AC, and is found by this proportion BD : BE : : BA : BF, or CD : CE : : CA : CE.*

1. It is required to cut off 2 acres from the triangle ABC of 6 acres, by a line drawn from D, 230 yards from B; the line BC being 466 yards, and BA 420.



Ans. 6 : 2 : : 466 : 155$\frac{1}{3}$ = BE, and 230 : 155$\frac{1}{3}$: : BA 420 : BF 283$\frac{15}{23}$; then DF is the dividing line.

If E had fallen between D and C, then AC must have been divided.

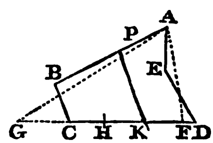
2. Divide the triangle ABC, of which the sides are AB 451, BC 528, and AC 364 links, in the ratio of 7 to 9, by a line drawn from D in BC, 363 links from B.

Ans. BF in AB 287 links.

3. Divide the triangle ABC, of which AB is 464, and BC 580 feet, and the angle ABC 64°, in the ratio of 3 to 5, by a line drawn from E in AB, 290 feet from B.

Ans. BF in BC 348 feet from B.

Prob. XXXVI. To divide any field ABCDE in a given ratio, by a straight line drawn from the point P in AB, one of its sides.



Reduce the field to the triangle AFG, by Prob. XXXIV. of Practical Geometry, having its base in the side CD, which the dividing line will cut. Divide the triangle AFG in the given ratio by the line AH, by Prob. XXX. Draw AK parallel to PH, and join PK: it will be the dividing line.†

Note 1. If the point K fall in CG, the field must be reduced to a triangle whch has its base in BC, or a triangle equal to PCK must be made by a line drawn from P to BC.

Divide the quadrilateral ABCD, of which the sides are AB 255, BC 284, CD 313, and AD 472 yards, and the angle ABC 57°, in the ratio of 6 to 7, by a line drawn from P in AD, 118 yards from A. Ans. BH 294, and BK 190·06 yds.

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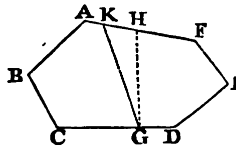
* Join DA, and draw EF parallel to it, the triangles BAD, BFE, are evidently similar; wherefore BD : BE : : BA : BF.

† Draw the parallels HP, KA, and join HA, intersecting PK in the point O. Since the triangles KAP, KAH have the same base KA, and lie between the same parallels KA, HP, they are therefore equal (El. Geom. 15, Cor.) Now, if from these equals we take away the common part KOA, there will remain POA = KOH, whence the line KP divides the figure in the proportion required.

NOTE 2. As the method of dividing the field geometrically by parallels is much easier than the arithmetical, it is best to do it in that way very accurately, and then to measure the result by the scale.

PROB. XXXVII. From a given field ABCDEF, to cut off any quantity by a straight line drawn from the point G in the side CD.



Draw GH as near to the position of the line required as you can conjecture, and calculate the area of the space ABCGH. Then, if this differs from the quantity to be set off, divide the difference by $\frac{1}{2}$ GH, the guess-line, the quotient is the length of a perpendicular, to be set off on either side of GH according as the quantity is too great or too small.

It is required to set off from the point G, in the side of the field ABCDEF, 8 acres towards BC.

Draw GH by guess, and measure the space GHABC, which suppose $= 9{\cdot}0496$ acres, or $1{\cdot}0496$ acres too much; now if GH $= 728$ links, then $10496 \div 364 = 288$, the perpendicular to be set off from H towards BC, and the line GK drawn through this point is the dividing line required.

When a quantity of land, such as a common, is to be divided among several proprietors in certain proportions, the quantity to be assigned to each will be as the value of his claim, divided by the quality or value of the ground allotted to him. This may be done by adding into two sums the contents and the values: then, by distributive proportion or fellowship, compute the value of each person's share; and from the quality of the ground where his share is to be determined, find what quantity will amount to the value of his share, and lay it off by the last problem.

Suppose it were required to divide 780 acres among three proprietors, whose estates are £1000, £3000, and £4000 a-year, and the values of the land in which their shares are to lie are 5s., 8s., and 10s. the acre respectively.

The claims being as 1, 3, and 4, and the qualities as 5, 8, and 10, the quantities assigned to them must be as $\frac{1}{5}$, $\frac{3}{8}$, and $\frac{4}{10}$, or as 8, 15, and 16, and their shares 160, 300, and 320 acres.

PROB. XXXVIII. To transfer, and to enlarge or diminish a plan.

After the first plan is completed, it will be necessary to draw out a fair one upon vellum or paper.

There are various ways of doing this.

I. If the fields be generally bounded by straight lines, lay the plan upon the clean paper, keeping it firm by weights, and prick through all the corners of the plan, and then connect the points on the clean paper.

II. Lay a piece of paper covered with black-lead dust between the papers, with the powdered side towards the clean paper, and with a blunt needle trace all the lines on the rough plan with such pressure that the impression may reach the clean paper; after which they are to be traced with ink upon the clean paper.

III. Divide the rough plan into small squares, and divide the paper to which it is to be transferred into as many squares; then copy the parts of the plan found in each square into the corresponding square of the other plan. In this manner the plan may be enlarged or diminished in any proportion, by making the squares in that proportion.

IV. There are several instruments useful for transferring, enlarging, and diminishing plans, as the proportional compasses, the pentagraph, and the eidograph.

A plan may be enlarged or diminished in any proportion on the first paper, by Prob. XXXVII. of PRACTICAL GEOMETRY, and afterwards transferred to the clean paper by any of the preceding methods.

After the plan is copied upon the clean paper, write such names, remarks, or explanations as are reckoned to be necessary; draw a meridian line with a fleur-de-lis pointing to the north, and in a convenient corner lay down a scale for measuring the parts of the plan. The title of the plan must be placed in a conspicuous part, and properly ornamented. After which, every part must be coloured or illuminated in the way that appears most natural. Rivers, woods, hills, hedges, houses, roads, &c. must all be distinguished by proper representations. But these things require to be learned by practice.

TO FIND THE WEIGHT OF CATTLE.

TAKE the girt close behind the shoulder, and the length from the fore part of the shoulder-blade along the back to the bone at the tail, which is in a vertical line with the buttock, both in feet. Multiply the square of the girt by 5 times the length, and divide the product by 21; the quotient is the weight, nearly, of the four quarters, in imperial stones of 14 lbs. avoirdupois.*

NOTE. The four quarters in very fat cattle will be about $\frac{1}{20}$ more, and in very lean cattle $\frac{1}{20}$ less than the weight obtained by the rule. The four quarters are very little more than half the weight of the living animal; the skin weighs about the 18th part, and the tallow about the 12th part of the whole.

1. What is the weight of the four quarters of an ox, of which the girt is 6 feet 6 inches, and the length 5 feet 10 inches?

Ans. $6{\cdot}5^2 \times 5$ ft. 10 in. $\times 5 \div 21 = 58$ stones 10·8888 lbs.

2. What is the weight of the quarters of a sheep, of which the girt is 3 feet 1 inch, and the length 2 feet 8 inches?

Ans. 6·036 stones.

3. What is the weight of a hog which is 4 feet 6 inches in girt, and 3 feet 4 inches in length? Ans. $16\frac{1}{14}$ stones.

4. What is the weight and value of an ox measuring $6\frac{1}{2}$ feet in girt, and $5\frac{3}{4}$ feet in length, at 11s. 6d. a-stone, sinking offals? Ans. 57·8423 stones, value £33·2593.

5. What was the value of the four quarters of the Dunearn ox, which measured 9 feet $3\frac{1}{2}$ inches in girt, and 5 feet $7\frac{1}{2}$ inches in length, at 10s. 6d. a-stone? Ans. £60, 14s. $1\frac{1}{4}$d.

6. What is the weight of the four quarters of a calf measuring 3 feet in girt by $2\frac{1}{4}$ feet in length? Ans. $4\frac{23}{28}$ stones.

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* It has been found by experiment, that the weight of a bullock divided by the product of the square of the girt behind the shoulder-blade into the length from the shoulder-blade to the buttock is $= 3\frac{1}{3}$ lbs. avoirdupois $= \frac{5}{21}$ of an imperial stone. Hence the weight of all cattle whose specific gravity is nearly alike will be obtained by the rule.

TO FIND THE WEIGHT OF A STACK OF HAY.

To the height from the ground to the eaves add half the height from the eaves to the top; then multiply the sum, and the length and breadth of the stack, into one product, all of them being taken in feet. Divide the product by 27, to bring it to yards. This, multiplied by 6, will give the number of stones, if the hay be new; but if the stack has stood a considerable time, add a third to it; or if it be old hay, add a half to it.

NOTE. If the form of the stack resemble any of the figures in MENSURATION OF SOLIDS, its content in cubic yards may be found by the rules there given, and its weight found as in the rule.

1. How much hay does a new stack contain, of which the length is 25 feet, the breadth 9 feet, the height from the ground to the eaves 14 feet, and above the eaves 8 feet?

Ans. $18 \times 25 \times 9 \times 6 \div 27 = 900$ stones.

2. How much old hay in a stack 40 feet long and 16 feet broad, the height to the eaves 15 feet, and above 8 feet?

Ans. $4053\frac{1}{3}$ stones.

3. How much new hay in a stack 50 feet long and 30 feet broad, the height to the eaves 20 feet, and above 14 feet?

Ans. 9000 stones.

4. How much hay in a stack which has stood 4 weeks 60 feet long and 35 feet broad, the height to the eaves 24 feet, and above 16 feet? Ans. $19911\frac{1}{9}$ stones.

THE WORKS OF ARTIFICERS.

Artificers take the dimensions of their work with a measuring-line, divided into feet and inches, or by the carpenter's rule, or by a yard divided into inches and parts.

The work is generally computed by duodecimal multiplication, in which the inch is supposed to be divided into 12 parts, and each part into 12 seconds, &c.

Rule. Multiply each denomination of the multiplicand by the feet of the multiplier, and place the product under that denomination of the multiplicand from which it arises, carrying at 12. Then multiply by the inches of the multiplier, and set each product a denomination farther towards the right hand. Next multiply by the parts, if any, and set the products a place still farther to the right. Then add the products.

1. Multiply 9 f. 4 in. by 3 f. 8 in.

3	8		
28	0		
6	2	8	
34	2	8	product.

2.	Multiply	98	3		by	5	6		........Ans.	540	4	6			
3.	....	148	3		by	8	9		....	1297	2	3			
4.	....	87	6	8	by	11	10		....	1036	0	10	8		
5.	....	63	4	6	by	8	9	6	....	557	2	0	9		
6.	....	55	8	7	by	72	6	3	....	4040	6	2	7	9	
7.	....	105	3	4	by	27	9	6	....	2925	10	1	8		
8.	....	208	7	9	by	12	5	4	....	2596	5	9	4		
9.	....	365	11	8	by	13	6	3	....	4948	2	11	11		
10.	....	185	10	9	by	15	9	8	....	2938	2	2	11		

Note. The feet in the product are square feet, 9 of which make a square yard, and 36 square yards make a rood of building. The inches in the product are 12th parts of a square foot, or each of them is 12 square inches, and the parts are square inches. The lower denominations are commonly expressed in fractions of a square inch: thus, 8 seconds are $\frac{2}{3}$ of a square inch, 9 seconds are $\frac{3}{4}$, and 7 seconds 6 thirds are $\frac{5}{8}$.

OF THE CARPENTER'S SLIDING RULE.

The works of artificers, as well as the quantity of timber, are often computed by the sliding-rule.

This rule consists of two pieces, each a foot long, fastened together with a folding joint, with a slider in one of the pieces.

The edge of each piece of the rule is divided into 10 equal parts, and each part is subdivided into 10 equal parts; so that by it the dimensions may be taken in feet and decimals.

One of the faces is divided into inches, and 8th or 10th parts; and on the same face are several plane and diagonal scales, the diagonal being divided into 12 parts.

On the other face, the piece which has the slider contains four lines, two on the slider marked B and C, and two on the rule; one under the slider marked A, and the other above it marked D. The three lines A, B, and C, are of the same length, and divided in the same way: the divisions on D are double of those on the other lines. These divisions are all logarithmical; that is, if the distance between the first 1 and the other 1 be divided into 1000 equal parts, the distance between 1 and 2 is 301 parts, which is the logarithm of 2, and the distance between 1 and 3 is 477, the logarithm of 3, &c.

The first 1 may be read 1, or 10, or 100, and all the rest are valued according to it. If it be read 1, the second 1 is 10, and the third 1 is 100, and then the first 2 is read 2, and the second 2 is 20; but if the first 1 be called 10, the second 1 is 100, and the third 1000, and then the first 2 is 20, and the second 2 is 200. And all the other divisions and subdivisions are valued in the same way.

On the same face of the rule, there is on the other piece of it a table of the value of a load, or of 50 cubic feet of timber, at all prices, from 6d. to 2s. each foot.

PROB. I. To multiply numbers by the rule.

Set 1 on B opposite to the multiplier on A; then opposite to the multiplicand on B will be the product on A.

1. Multiply 16 by 6..............Ans. 96.
2. .... 23 by 14....................322.
3. .... 27 by 23....................621.
4. .... 68 by 46..................3128.

PROB. II. To divide numbers.

Set the divisor on B to 1 on A; then against the dividend on B will be found the quotient on A.

1. Divide 96 by 24..................Ans. 4.
2. .... 576 by 48........................12.
3. .... 156 by 23........................6·8.
4. .... 988 by 76........................13.

Prob. III. To work a proportion.

Set the first term on B to the second on A; then against the third on B will stand the fourth on A.

1. Required the fourth proportional to 12, 28, and 114. Ans. 266.
2. Required the third proportional to 18 and 54. Ans. 162.
3. If 14 men build 4 roods, how many will in the same time build 28 roods? Ans. 98 men.
4. If 42 men perform a piece of work in 108 days, in what time will 72 do it? Ans. 63 days.

Note. This, with the two preceding rules, depends upon this principle: In a proportion, the difference between the logarithms of the first and second terms is equal to the difference of the logarithms of the third and fourth; and 1 is to the multiplier or divisor, as the multiplicand or quotient is to the product or dividend.

Prob. IV. To extract the square root.

Set 1 on C to 1 on D; then against the given number on C is its square root on D.

Note. The 1 on C must be read 1, or 100, or 1000; and the 1 on D must be read 1, or 10, or 100 accordingly.

1. Required the square root of 576..............Ans. 24.
2. .... of 196...................14.
3. .... of 4096...................64.
4. .... of 9216...................96.

Prob. V. To find a mean proportional between two numbers.

Set the less on C to the same number on D; then against the greater number on C will stand the mean proportional on D.

1. Required the mean proportional between 4 and 36. Ans. 12.
2. .... 144 and 576. 288.
3. .... 513 and 57. 171.
4. .... 128 and 32. 64.

TO MEASURE TIMBER.

Prob. I. To find the area of a board.

Rule. Multiply the length by the mean breadth.

Note. When the board tapers regularly, half the sum of the breadths at the ends is the mean breadth.

By the Sliding-Rule.

Set the breadth in inches on A to 12 on B; then against the length in feet on B will be the content on A, in square feet and decimals.

1. Required the content of a board 12 feet 6 inches long, and 1 foot 3 inches broad. Ans. 15 feet 7 inches 6 parts.
2. Required the content of a board 13 feet 4 inches long, and 1 foot 8 inches broad. Ans. 22 feet 2 inches 8 parts.
3. Required the content of a board 11 feet 10 inches long, and 11 inches broad. Ans. 10 feet 10 inches 2 parts.
4. Required the content of a board 16 feet 9 inches long, and 2 feet 2 inches broad. Ans. 36 feet 3 inches 6 parts.
5. Required the content of a board 14 feet 11 inches long, and 9 inches broad. Ans. 11 feet 2 inches 3 parts.
6. Required the content of a board 10 feet 10 inches long, and 8 inches broad. Ans. 7 feet 2 inches 8 parts.

Prob. II. To find the content of squared or four-sided timber.

Rule. Multiply the mean breadth by the mean thickness: the product, multiplied by the length, will give the content.*

Note. If the tree tapers regularly from the one end to the other, take the mean breadth and thickness in the middle, or take half the sum of the dimensions at the two ends for the mean dimensions. If it does not taper regularly, take several dimensions at equal intervals, and divide their sum by the number of them for the mean dimensions; or divide the tree into several convenient lengths, find the content of each piece separately, and their sum will be the whole content.

By the Sliding-Rule.

Find a mean proportional between the breadth and thickness. Then set the length on C to 12 on D; and against the

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* Sometimes $\frac{1}{4}$ of the circumference of the tree is used as the side of the mean square, and this multiplied by itself and by the length is accounted the solid content. This method is, however, very erroneous, always giving the content too great; and supposes the quarter-girt to be an arithmetical instead of a geometrical mean proportional between the breadth and thickness. The content of the log, in the second example, by this method would be 64 feet 2 inches $0\frac{3}{4}$ part, or 2 feet 3 inches $6\frac{3}{4}$ parts too great.

mean proportional on D in inches will be the solid content in feet on C.

NOTE. If the mean proportional be in feet, use 1 instead of 12 on D.

1. Required the content of a log, the length 24 feet 6 inches, mean breadth 1 foot 1 inch, and mean thickness 1 foot 1 inch.
Ans. 28 feet 9 inches $\frac{1}{2}$ part.

2. Required the content of a log, the length 27 feet, mean breadth 1 foot 10 inches, and mean thickness 1 foot 3 inches.
Ans. 61 feet 10 inches 6 parts.

3. Required the content of a log, the length 18 feet 6 inches, mean breadth 1 foot $4\frac{1}{2}$ inches, and mean thickness 1 foot 2 inches.
Ans. 29 feet 8 inches $1\frac{1}{2}$ part.

4. Required the content of a log, the length 20 feet 6 inches. mean breadth 1 foot $2\frac{1}{2}$ inches, and mean thickness 1 foot $2\frac{1}{4}$ inches.
Ans. 29 feet 11 inches $2\frac{1}{8}$ parts.

5. Required the content of a log, the length 30 feet 8 inches, mean breadth 2 feet 1 inch, and mean thickness 2 feet 2 inches.
Ans. 138 feet 5 inches $1\frac{1}{3}$ part.

6. Required the content of a log, the length 40 feet 7 inches, mean breadth 2 feet 3 inches, and mean thickness 1 foot 9 inches.
Ans. 159 feet 9 inches $6\frac{3}{4}$ parts.

PROB. III. To find the content of round timber.

COMMON RULE. Take one-fourth of the mean girt, and square it, and multiply it by the length for the content.

By the Sliding-Rule.

Set the length in feet on C to 12 on D; then against the quarter-girt in inches on D will be the content in feet on C.

NOTE 1. In order to reduce the tree to such a circumference as it would have without its bark, $\frac{1}{4}$ of an inch should be deducted from the quarter-girt when the thickness of the bark is $\frac{1}{4}$ of an inch; but when the bark is thicker, which is rarely the case, a little more than $\frac{1}{4}$ of an inch must be allowed. No rough timber under 6 inches diameter is accounted measurable.

NOTE 2. The common rule gives the content too small, by 3 feet on every 11 feet of content; yet it is universally used in practice, being originally introduced in order to compensate the purchaser of round timber for the waste occasioned by squaring it.*

* Let l = the length, and c = the mean circumference; then the rule is $\left(\frac{c}{4}\right)^2 \times l = \cdot0625c^2l$; but the content of the cylinder is $= \cdot0795775c^2l$; hence the content, as given by the rule, is to the true cylindrical content, as $\cdot0625 : \cdot0795775$, or as $11 : 14$ nearly.

RULE II. Take one-fifth of the girt, and square it, and multiply by twice the length for the true content nearly.*

By the Sliding-Rule.

Set twice the length on C to 12 on D; then against $\frac{1}{5}$ of the girt on D will be the content in feet on C.

1. Required the content of a piece of round timber $9\frac{1}{2}$ feet long, and its mean girt 14 feet.

Ans. 116 feet $4\frac{1}{3}$ inches by the common rule; or, true content 148 feet 11·52 inch by Rule II.

2. Required the content of a tree 24 feet long, and its girts at the ends 14 and 2 feet.

Ans. 96 feet by the common rule; the true content is 122·88 feet.

3. How much timber in a tree 18 feet long, and its mean girt 5 feet 8 inches?

Ans. Common rule 36 feet $1\frac{1}{2}$ inch; true content 46 feet 2 inches 10·56 parts.

4. How much timber in a tree 32 feet long, its girts in the middle of every 8 feet being 64, 56, 52, and 46 inches?

Ans. 41 feet $10\frac{1}{8}$ inches by the common rule; true content 53 feet 6 inches 9·28 parts.

5. Required the content of a tree 30 feet long, the girts in the middle of every 10 feet being 50·4, 54·8, and 60·8 inches.

Ans. 40 feet 1 inch 2·9 parts by the common rule; true content 51 feet 3 inches 11·872 parts.

6. Required the content of a tree 55 feet long, the girts in the middle of every 11 feet being 72, 56, 42, 35, and 25 inches.

Ans. 56 feet 11 inches $8\frac{5}{8}$ parts by the common rule; true content 72 feet 11 inches 1·92 part.

7. Required the content of a tree 50 feet long, its mean girt being 7 feet.

Ans. 153 feet $1\frac{1}{2}$ inch by the common rule; true content 196 feet.

8. Required the content of a tree 48 feet long, the girts at its ends being 60 and 18 inches.

Ans. 31 feet $8\frac{1}{4}$ inches by the common rule; true content 40 feet 6·72 inches.

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* The solidity of the cylinder is $\cdot 0795775c^2l$, and that by the rule is $\left(\frac{c}{5}\right) \times 2l = \frac{c^2}{25} \times 2l = \frac{2}{25}c^2l = \cdot 08c^2l$; hence the content, as given by the rule, is to the true content as $\cdot 08 : \cdot 0795775$, or as $1 : \cdot 99472$; hence the rule gives 1 foot too much in 190 feet of content.

9. Required the content of a tree 45 feet long, the mean girt being 74 inches.

Ans. 106 feet 11$\frac{7}{18}$ inches by the common rule; true content 136 feet 10·8 inches.

10. Required the content of a tree 17$\frac{1}{4}$ feet long, the girts in five different places being 9·43, 7·92, 6·15, 4·74, and 3·16 feet.

Ans. 42·5195 feet by the common rule; true content 54·424992 feet.

PROB. IV. To calculate the value of roods, yards, feet, inches, &c. at any number of shillings and pence per rood.

RULE. Bring the roods, yards, feet, inches, &c. into a state of duodecimals; thus,

Write down in the same line with the roods one-third of the yards, and if there is any remainder, for each unit of it add 9 feet to the given feet, inches, &c., then annex four-ninths of this sum to complete the multiplicand, and multiply it by the shillings and pence, as in duodecimal multiplication. The highest or left-hand number of the product is shillings, the second pence, and the third twelfth-parts of a penny, &c.*

NOTE. Instead of taking $\frac{4}{9}$ of the feet and inches, add $\frac{1}{3}$ to them, and then take $\frac{1}{3}$ of the sum.

Find the value of 12 roods 15 yards 4 feet 3 inches, at 25s. per rood.

Here we write down 12 on the left, and next to it the third part of 15, or 5, and as there is no remainder, we next write down $\frac{4}{9}$ of 4 feet 3 inches, or 1 : 10 : 8, and the complete multiplicand is

	12	5	1	10	8 × 25 = 5 × 5
					5
	62	1	9	5	4
					5
2,0)	31,0	8	11	2	8
	£15	10	9 very nearly.		

Find the value of 15 roods 16 yards 7 feet 3 inches, at 31s. 6d. per rood.

* The editor is indebted for this simple and useful rule to Mr Duff, surveyor, Edinburgh.

Here the prepared multiplicand is

15 5 7 2 8 to be multiplied by $31\frac{1}{2} = 3 \times 10 + 1\frac{1}{2}$.

15	5	7	2	8
				3
46	4	9	8	0
				10
464	0	0	8	0
15	5	7	2	8
7	8	9	7	4
487	2	5	6	0

$= £24$, 7s. $2\frac{1}{2}$d. very nearly.

1. Find the value of 6 roods 5 yards 6 feet 3 inches, at 21s. 4d. per rood. Ans. £6, 11s. $4\frac{1}{2}$d.
2. Find the value of 35 roods 27 yards 4 feet 8 inches, at 27s. $9\frac{1}{2}$d. per rood. Ans. £49, 13s. $11\frac{1}{2}$d.
3. Find the value of 56 roods 17 yards 5 feet 4 inches, at 15s. $10\frac{1}{2}$d. per rood. Ans. £44, 16s. $9\frac{1}{11}$d.
4. Find the value of 47 roods 19 yards 7 feet 6 inches, at 34s. $8\frac{3}{4}$d. per rood. Ans. £82, 11s. $4\frac{3}{4}$d.

MASON WORK.*

RUBBLE WORK is measured in three different ways.

I. When the tradesman furnishes all materials.

Find the depth of the foundation at several places, and take the mean height from the foundation to the top of the side-walls. Take the length of the side-walls on the outside, and the breadth of the gables or cross-walls on the inside of the building.

Gable-ends are measured by multiplying the height from the level of the side-walls to the bottom of the chimney-stacks by half the sum of the breadths at the top of the side-walls and at the bottom of the chimney-stack; and the chimney-stack is measured by multiplying half the girt by the height from the bottom of the stack to the top of the cope.

Chimney-flues are measured by the lineal foot, from the top of the stack to the bottom of the jambs.

Dormer-windows on side-walls are measured by adding the thickness of one haunch to the length of the square part, and

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* The rules for the Mensuration of Artificers' Works, with the various allowances, have been furnished by an eminent surveyor in Edinburgh, and cannot fail to be of great advantage to the students for whom this section is intended. The allowances apply principally to Scotland; but the rules for taking the dimensions are also applicable both to England and Ireland.

multiplying it by the height from the level of the side-walls to the bottom of the angle; and the angular part and stack are measured the same way as a gable-end and chimney-stack.

All projections, whether external or internal, if they do not exceed 2 feet, are found by adding one return to the length, and multiplying the sum by the height and thickness, and reducing it to the standard of the wall.

An allowance for workmanship of 1 foot by 9 inches, multiplied by the length, is made for every levelling for joists and belts in rubble walls; and when walls are 2 feet thick or more, 1 foot by the thickness of the wall, multiplied by the length, is made for levelling the tops of side-walls, skews, and chimney-stacks; and when under 2 feet thick, the thickness by its half, and by the length, is allowed; but no allowance is made for belts on ashlar fronts.

An allowance of 9 inches square by the length is made for levelling for bond timbers and ragulates for roofs in the chimney-heads only; 1 foot by 9 inches is allowed for ragulates left for stairs; and 1 foot by 6 inches for thin walls. These allowances must all be reduced to the standard of the walls in which they are made, and rated as workmanship only.

The daylight or net opening of all apertures is to be deducted.

Rough stones more than 3 feet in length, placed as safes over voids, are to be taken by number, according to their different lengths.

Arches over cellars, &c. are taken by the net average girt, and by the length and by the depth of the arch-stones for the thickness, and are double measure; and arches having been included in the general dimensions are to be again taken by their height, thickness, and length, and reduced to the standard of the wall.

All upright circular walls are double measure, if not above 2 feet thick, and if above that thickness single measure, and a 2 feet wall *extra* added; and walls circular on one side only are allowed 1 foot thick round the circular part as double measure, and reduced to the standard of the wall, besides the solid content of the straight part.*

The rubble of stair-steps and platts is taken by their length without the wall, and by their breadth and thickness, and in all cases reduced to 1 foot thick.

Rubble is allowed for pavement laid on lime, and in no case is the thickness reckoned less than 4 inches.

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* The double measure for circular walls is understood to be far too great an allowance, except when the circle is of small diameter and the work well executed.

In measuring separated pillars, when the face or front of the pillar does not exceed 5 feet in length, they are taken by their net height and length, and an allowance of 2 feet square by the height is made for carrying up the scontion. But this allowance applies only to pillars at and above 2 feet thick: all below that have the net thickness added to the length.

II. When the tradesman furnishes workmanship only.

The dimensions are taken over both side-walls and gables, and no deduction is made for apertures.

III. When the tradesman furnishes workmanship, lime, and sand.

The outside-walls are measured by including the thickness of one side-wall, and one-half of the apertures is deducted.

NOTE. Rubble walls, in all the three cases, at and below 18 inches thick, are to be reduced to 1 foot, and all above 18 inches reduced to 2 feet thick, and measured by the rood of 36 square yards.

On doors and windows where there is no hewn work, an allowance is made of 1 foot square by the length, in name of hammer-dressed scontions.

HEWN WORK.

Hewn Work in Rubble Walls. The rybats of doors and windows are measured by girting from the bottom of the check outward, including the backset, if any. Sills and lintels are taken for the length over the face of the rybats, including the projection of one end, if projected; and the girt is taken as in the rybats.

Hewn corners are taken by the height for the length, and by the mean girt for the breadth.

Skews are taken by the length and by the girt, and chimney-copes by the extreme length all round, and for the breadth by girting from the open of the flue down to the chimney-stack.

When the whole front of a building is of hewn or polished work, it is taken by the extreme length and height of the different species of work, including the sides of projections, if any; but no allowance is made for the internal corners of such projections. All apertures are deducted; but the breasts and checks of rybats, together with the under bed and checks of the lintel, and upper bed of the sill, including their rests, are measured and added.

When architrave rybats are placed in a hewn front, the deductions are taken over these; and such moulded architrave rybats are measured by the height, and by girting from the bottom of the check outwards to the face of the plain ashlar.

The lintels are girted in the same manner, and the length is taken round the ends.

Moulded architrave rybats of main doors, or otherwise, are taken in the same manner, and the whole reported as moulded work, excepting when plain ashlar stones are placed in the reveals, between the outband rybats and the checks; in which case these must be deducted, and added to the plain ashlar.

The Hewn Work of Arches is measured by finding the mean-height of the arch stones, and for the length by laying the line round the middle of the face of the arch. The soffit and check are taken for the length round the check, and for the breadth by girting from the bottom of the check outward to the face of the arch. Both face and soffit are reckoned double measure. Arches in upright circular walls are allowed three measures.

When pannels are sunk on ashlar work, after they are included in the surface, the sunk part, and that round the edges, are taken over again; but if a moulding is round it, the whole is taken as moulded work, and not included in the plain surface.

All hewn work cut circular for skews is allowed 6 inches by the length for cutting.

Rustic work, whether square or champhered, is first measured superficially, and the checks or champhers are measured over again. Giblat checks, in like manner, are measured over again, after having been included in the face on scontions.

Pilasters, when they are raised out of the solid stones, and built in courses along with the ashlar, are girted in along with the ashlar, and the sunk part and edges are taken over again. If the pilasters are fluted, they are measured over again as moulded work, girting into the flutes and over the fillets. The cabled part, if any, is measured in the same way, and allowed double measure. The bases and capitals are girted as mouldings.

NOTE. The measuring over again of pilasters ought to be only for workmanship. But a better way is to measure single as mouldings.

Columns, of which the shafts are diminished with a curve or swell, are allowed double measure and a half; and if the neck-moulding is wrought on the shaft, they are allowed three measures. When the shafts are diminished straight, without a swell, double measure is only allowed, and a half more if the neck-moulding is wrought on the shaft. The fluted and cabled parts of columns are measured the same as in pilasters, after they are taken for plain work, as above. The bases and capitals are girted as other mouldings, with the usual allow-

ance; and the number and size of carved capitals must be given.

Cornices are taken for their length at the extremities of their greatest projection, and for their breadth by girting their mouldings; and so much of the superficies of the upper bed as is without the wall is added.

Block and dentil cornices, after being measured in the same manner, have the backs and soffits of the recesses, together with the ends of the blocks and dentils, added.

Dentils are, however, generally measured lineally and not as surfaces.

The steps of hanging stairs, whether moulded or plain, are girted at their mean breadth, including both joints, and for their length what is seen, including 6 inches of rests in the wall. The soffit and ends of wheel steps are taken over again, so far as is without the walls; and the ends of both square and wheel steps are taken at their extreme breadth and depth.

The joints of platts, if joggled, are also taken.

The skirting of hanging stairs is taken by the extreme length and breadth of the stones, including the upper edge.

The steps and platts of newel stairs are taken by girting at the mean breadth, allowing 1 inch of overlap on each step, and for the length by what is seen, allowing 6 inches on each end for rests. The newels are girted round, including the backset. The tails are taken as scribbled work, and the soffits of steps according to the kind of work upon them.

Pyramids or obelisks, if they are built in courses, are girted for the length at the bottom of each course, and between the joints for the breadth. When they are made of one stone, they are girted for the length at the bottom, and for the breadth by the sloping height; and if they are polygonal figures, the peends or angles are generally allowed about 3 inches broad on each angle over and above the net girts.

In measuring curb-stones, besides the upper bed, 6 inches are allowed on the edge, of the same work with the upper bed.

Hewn work of every kind, as well as coursed, hammer-dressed, or scribbled work, is measured by the superficial foot.

NOTE. In measuring rough-casting, the whitewashing on the faces and breasts of rybats, belts, chimney-copes, &c. is taken as rough-casting.

1. How much rubble work of the standard thickness of 2 feet is in a house of 3 stories, 60 feet long and 30 broad within walls, the height 30 feet from the foundation to the top of the side-walls, 12 more to the foot of the chimney-stacks, which are 7 feet high, 10 broad, and 3 thick, the skews are

14 feet 6 inches long, the side-walls $2\frac{1}{2}$ feet thick, and the end-walls 3 feet thick, with two doors, each 7 feet by 4 feet, 22 windows, each 5 feet by 3 feet, and 12 windows, each 4 feet by $2\frac{3}{4}$ feet?

A side-wall,	$66\times30\times1\frac{1}{4}$	$=2475$ sq. feet.
An end-wall,	$30\times30\times1\frac{1}{2}$	$=1350$
A gable-end,	$\frac{1}{2}(35+10)=22\frac{1}{2}\times12\times1\frac{1}{2}$	$=405$
A chimney-stack,	$10+3=13\times7\times1\frac{1}{2}$	$=136\frac{1}{2}$
		$4366\frac{1}{2}$
		2
		8733
Levelling side-walls,	$60\times2\times2\frac{1}{2}$	$=300$
Ditto for joists,	$60\times\frac{3}{4}\times2$	$=90$
Ditto 4 skews,	$14\frac{1}{2}\times4\times3$	$=174$
Ditto chimney-tops,	$10\times2\times3$	$=60$
		9357
2 doors,	$7\times4\times2=56$	
22 windows,	$5\times3\times22=330$	
12 windows,	$2\frac{3}{4}\times4\times12=132$	
	518	
Add $\frac{1}{4}$ for thickness,	$129\frac{1}{2}$	
		$647\frac{1}{2}$
Content 26 roods 31 yards $6\frac{1}{2}$ feet,		$=8709\frac{1}{2}$ sq. feet.

2. How much hewn work in 22 window-lintels and sills, each 4 feet by $1\frac{2}{3}$ foot; 12 lintels and sills, each $4\frac{1}{2}$ feet by $1\frac{1}{3}$ foot; two door-lintels and sills, each 5 feet by $1\frac{2}{3}$ foot; 22 pairs of rybats of 5 feet, 17 rybats of 4 feet, and 2 ditto of 7 feet, all of them 14 inches broad; skews 64 feet by 14 inches, coping of chimney-stacks 44 feet by 20 inches, and coping of the roof 60 feet by 16 inches?

Ans. $687\frac{1}{3}$ square feet.

3. What should be charged for the workmanship of a house of 2 stories, 36 feet long and 24 feet broad within the walls; the side-walls 2 feet thick and 24 feet high, measured from the foundation; the gables 3 feet thick and 16 feet higher than the side-walls to the bottom of the chimney-stacks, which are 7 feet broad, 3 deep, and 8 high; the skews are 19 feet 3 inches in length, and there are 110 feet of flues; the rubble work, reduced to the standard, is at £2 per rood, and the flues at 4d. per foot? Ans. £36, 5s. 11d.

4. A house of 3 stories is 45 feet long and 28 feet broad within walls, and the height from the foundation to the top of the side-walls is 30 feet; the gables rise 18 feet above the side-walls to the bottom of the chimney-stacks, which are 8 feet wide, 3 deep, and 10 high; the skews are 21 feet 11

inches long; the side-walls are $2\frac{1}{2}$ feet, and the gables are 3 feet thick; there are 2 doors in the sides, each $7\frac{1}{2}$ feet by 4 feet; 12 windows in the sides, and 6 in the ends, each 6 feet by $3\frac{1}{2}$ feet. Required the expense of the materials and workmanship of the rubble work, at £10, 6s. 8d. per rood, allowing £2, 14s. per rood for levelling the side-walls.

Ans. £234, 15s. 1d.

5. A house is 41 feet long, $20\frac{1}{2}$ feet broad within the walls, and 18 feet 9 inches high from the foundation to the top of the side-walls, which are 2 feet thick; the gables are $2\frac{1}{2}$ feet thick, and rise 8 feet 6 inches above the side-walls to the bottom of the chimney-stacks, which are 4 feet wide, $2\frac{1}{2}$ feet thick, and 5 feet 1 inch high. The broached hewn work consists of 4 skews, each 11 feet 6 inches by 1 foot 7 inches; 4 corners, each 18 feet 9 inches by $2\frac{1}{2}$ feet; and 2 chimney-stacks, the girt of each 13 feet, and the height 5 feet 3 inches. The droved hewn work consists of the rybats and lintels of 6 windows, each 13 feet 11 inches by 15 inches; 6 sills of ditto, each 3 feet 11 inches by 19 inches; the rybats and lintels of one window, 9 feet 3 inches by 15 inches; sill of ditto, $3\frac{1}{2}$ feet by 19 inches; the rybats and lintel of a door, $19\frac{1}{4}$ feet by 15 inches; sill of ditto, $4\frac{1}{4}$ feet by 19 inches; 3 pairs of jambs, each 6 feet by 2 feet; the lintels of ditto, each 4 feet 5 inches by 15 inches; 3 inner hearths, each 3 feet 1 inch by 18 inches; 3 outer hearths, each 3 feet 8 inches by 20 inches; kitchen jambs, 8 feet 8 inches by 2 feet 3 inches; lintel of ditto, 5 feet 8 inches by 15 inches; the hearth, 4 feet by 21 inches; and also $106\frac{1}{2}$ feet of flues. Required the content of the rubble work, and of the hewn work, and also the expense of the workmanship; the rubble work being at £3 per rood, the broached hewn work at 5d. per foot, the droved hewn work at 6d. per foot, and the flues at 6d. per foot.

Ans. 11 roods 1 yard 2 feet $4\frac{1}{4}$ inches rubble work; 396 feet 10 inches broached hewn work; 307 feet $5\frac{1}{4}$ inches droved hewn work. Expense of the whole £51, 14s. 5d.

BRICK WORK.

Brick Work is measured by the square yard, and reported as brick on edge or brick on bed, 9 inches or 14 inches thick; and all above that is reduced to 14 inches as the standard.

Brick walls are measured the same way as stone walls, and the net daylight of all apertures deducted.

Upright circular walls and arches are allowed measure and

half; and arches over apertures in upright walls are taken over again. Groin-arches are double measure; and 18 inches by the length and thickness allowed on the groin for cutting.

The tops of niches and spherical arches, whether of brick or stone, are allowed three measures.

When the skews on brick gables are feathered on edge or feathered and tongued, they must be taken over again; and in all cases $4\frac{1}{2}$ inches by the thickness of the skew above the thackgate is allowed for cutting. Chimney-stacks are taken by the height and by the breadth, adding the thickness of one haunch, if it does not exceed 18 inches; and in all above that thickness one-half of the haunch is added.

1. The sides of a brick vault are 18 feet long, 5 feet high, and 2 bricks thick; the girt of the arch 10 feet, and 1 brick thick; the end walls 8 feet long, 7 feet high, and $1\frac{1}{2}$ brick thick; the door 5 feet by $2\frac{1}{2}$ feet. How much does the vault measure at standard thickness?

Ans. 56·6516 square yards.

2. How many square yards of standard brick work in a wall 75 feet long, 15 feet 9 inches high, and 3 bricks thick?

Ans. 262 yards $4\frac{1}{2}$ feet.

3. A garden is 160 feet broad, and contains an imperial acre. Required the expense of enclosing it with a brick wall 10 feet 6 inches high, and $2\frac{1}{2}$ bricks thick, at 5s. $7\frac{1}{2}$d. per square yard of standard thickness, deducting 2 doors, each 6 feet 9 inches by 4 feet, and a gateway 11 feet wide.

Ans. £468, 0s. $11\frac{1}{4}$d.

CARPENTERS' AND JOINERS' WORK.

COMMON rough joisting is measured by adding to what is in sight the rests or hold of the joisting in the walls; and when that cannot be ascertained, an allowance not exceeding one foot on each end is made, and the content is estimated in square yards, stating the size and distance.

Framed joisting is measured in the same way for the scantling or bridged joists, and the surface-measure includes the beams. But beams and transoms are measured by the cubic foot. When joists are laid on the tops of walls, and the ends of couples joined to them, or when beam-fitted, and the wall plates fixed down to them,—in both cases they are taken as joisting.

Trussed and dressed beams are measured by the cubic foot, and the oak in trussed beams is reported lineal, stating the

size. Dwangs put between joists are classed with rough timber, such as safe-lintels, &c.

Deafening-boards are measured superficially; and when the composition is laid the hearth-places are deducted, and the boxing for the hearths stands as an equivalent for the floor.

Flooring is measured superficially, and reported according to its quality, as deal, or batten flooring, &c. No deduction is made for hearths where there is strong boxing under them not measured separately; but when that is the case, the hearths are deducted. When floors are cut to any angle or circle at or exceeding 45°, an allowance of 6 inches by the length is made for cutting. Hearth-borders are taken by the lineal foot.

Framed and bound roofing is measured by taking all the principal timbers that are connected with the main couples, by the cubic foot, and also the extra size of diagonals, when they are above 9 inches by 3 inches, and reported as cubic framed timber.

The surface of a square roof is measured by taking twice the depth from ridge to eave, and by the length from skew to skew.

A pavilion or hipped roof is taken by adding the length and breadth at the eaves to the length at the ridge, or to the length and breadth of the platform, and by the depth from ridge to eave. The platform is taken as flooring and joisting. A pavilion square roof, finishing in a point at the top, is taken by girting one end and one side at the eave for the length, and the depth, as before.

A conical or turnpike roof is measured by taking the circumference at the eaves, and by the slant height.

Eighteen inches by the length are allowed for each hip and valley. All openings for dormer-windows, skylights, and chimney-stacks are deducted, except when the opening is at or under 2 feet square; and when such deductions are made, 9 inches for the width of it are allowed at top and bottom, for bridling.

The contents of the wall-plates, including the sleepers built in for fixing them, are added to the surface-measure of the roof, unless when the wall-plates are above 2 inches thick; in which case they are taken as rough cubic timber.

The putting on of the iron-work of framed roofs ought to be included in the price; and, if furnished by the tradesman, charged by weight.

When there are two baulks in a common roof, the upper ones are included, and the under ones taken as joists, mentioning their size and distance.

Roofing and tile-lath are also measured superficially; and when sarking is put on slate eaves, it is measured as sarking only. Roofs upon circular walls are allowed double measure, and all domes three measures.

Roofs put upon polygons, when the scantlings are curved, are allowed double measure.

Battens for ridges and hips, whether square or rounded, and filleting for skews, are measured by the lineal foot, specifying the size.

Framing for brick partitions, if the standards are placed at regular distances, is measured superficially by the yard, as brick on edge, brick on bed, stating the distance of the standards; and when dressed door-standards are placed in such partitions, the apertures are deducted over the dressed standards.

When there are only a few detached standards in partitions, these are calculated to 3 inches square, and reported as rough standards; and the warpings, in that case, are to be reduced to 4 inches broad, and their thickness stated.

Standards for lath partitions are measured by the square yard, stating their size and distance, and deducting the doors over the door-standards, where dressed ones are placed. Runtrees at top and bottom (if any) must be reduced to 3 inches square, or 4 inches broad, stating the thickness.

Wall-battens for lath are measured by the superficial yard, and, if fixed with plugs, or iron holdfasts, are reported as such.

All standards set circular are allowed measure and half. Bond-timbers are taken by the lineal foot, stating the general size.

Lathing is measured by the square yard, and, when on circular walls, allowed measure and half. All arches and coves are allowed the same.

Domes and tops of niches are double measure, unless otherwise specified. An allowance of 6 inches by the length is made for cutting round circles and angles; and all apertures are deducted.

Dressed door-standards in brick or lath partitions are measured by their actual height, and, along with the lintels, reduced to 3 inches square.

All dressed posts or standards below 6 inches square are reduced to 3 inches square, and those at 6 inches square and above are reduced into cubic feet.

Dressed deal door-breasts or standards, not exceeding 8 inches broad and $1\frac{1}{4}$ inch thick, are reduced to 4 inches broad, and reported according to the thickness. All above 8 inches broad are reported by the superficial foot, as an

article by itself; and all above $1\frac{1}{4}$ inch thick are reduced to 3 inches square.

Grounds are measured by the lineal foot, specifying whether thick or thin, or if checked or grooved.

Sash-windows are allowed 2 inches more than the daylight for the height, and 3 inches for each side-facing more than the daylight for the breadth, when they are not more than 3 feet wide: all above that are allowed 1 inch on each side of the facing for every foot in width.

Windows with circular tops are allowed double measure for the circular part. Convex or concave windows are double measure; and, if made to fit an arch on the top, the arched part is taken at its extreme height and breadth, and allowed three measures. Flat segment-topped windows are allowed 9 inches for cutting; and, when the panes are square, are taken as windows without glass. Cupola lights with curved ribs, or astragals, are allowed three measures; but when straight, only two. Common skylight hatch-windows are taken by the net surface.

The sash-part of doors is measured by adding as much of the belt-rail to the height as the breadth of the stiles, and the remaining part is taken as bound work.

Chinese sash-lights are allowed double measure when the panes are of various figures, or circular; and if in a circular door, three measures are allowed; but only single measure when the panes are all one figure and one size.

Bound doors are measured by adding as many inches to the height as there are pannels in the height, and by the net breadth; and when the thickness is at or above $1\frac{3}{4}$ inch, double measure is allowed; below that thickness, when dressed on both sides, measure and half; but when dressed only on one side, no more than single measure.

Bound window-shutters are measured in the same way: if cut, two thicknesses are added to the length; and if checked for backfolds, the girt of one checked edge is added to the net breadth of both shutters.

Bound flush-and-bead shutters are measured by the square foot, specifying the thickness. Plain deal backfolds have the breadth of the cross heads added to the height, and are reported by the yard; and, if not more than 6 inches broad, they are reduced to 4 inches broad.

All circular bound work is allowed double measure.

Bound flush-and-bead doors, having two leaves, are measured like shutters with backfolds; but in shop-doors the sash is deducted from the bead-and-flush, and the sash and shutters taken by themselves.

Torus mouldings on bound work are taken by the lineal foot.

Plain deal-backed work, double deal doors, &c., are taken by the square foot; and if beaded on the joints, it should be specified, as well as the thickness.

Common plain deal, if dressed on both sides, is allowed measure and half, and reported by the square yard, stating the thickness; and whenever beads are put on the joints, half an inch is allowed in the measure for each bead.

Bound dado-lining is allowed, on the length, one inch for every external corner for nailing, and an inch of cover for every architrave; and on the height, besides an inch for the pannels, 3 inches more than the net measure between the base and surbase, and no notice taken of the stile-ends.

Plain dado and window linings, when done in a superior manner, are reported as such by the yard, stating the thickness, and the bars behind are included in the price.

Shelving in general is taken by the yard, stating the thickness. When cut circular, the net area is taken, and an allowance of 3 inches on each edge for cutting; and when circular on one edge, an allowance of 6 inches is made for cutting. When shelves are wrought on both edges, they are allowed measure and half; and grooves for shelves are reported by the lineal foot.

Plain deal work, dovetailed, is measured by the superficial foot, stating the thickness and quality; and all broad plain deal work, of whatever description, above 1½ inch thick, is taken by the square foot, and the thickness stated.

Mouldings are taken by their greatest length, and for their breadth by girting over the mouldings, allowing an inch more than is seen on base-mouldings for a rest on the plinth, and another allowance of one foot for every mitre more than four on base and surbase in one room.

The blocks on which architraves are set are included in the height of the architraves, and then taken over again as skirting, along with the base-plinth.

Cornices of doors and chimney-pieces are taken at their greatest projection for the length, and by girting the moulding for the breadth; the upper bed being taken as moulded work as far back as the projection, the remaining part to be of plain deal, if there be any.

The frieze-board is taken as plain deal, by the square foot, including what is behind the cornice; but when the frieze is under 6 inches broad, with an astragal at the bottom, the whole is taken as mouldings.

All mouldings, except small single ones, are estimated by

the superficial foot; dentils, Doric bells, &c. by the lineal foot.

The shafts of plain pilasters are taken by their extreme height and breadth, and estimated by the square foot, stating the thickness, and both edges are girted on the face. Fluted pilasters are taken in the same way, girting over the fillets, and into the flutes; and if the edges or returns are fluted, they are also girted in; but if they are planted returns, not fluted, they are taken as plain work, when above 2 inches broad. Cabled or reeded pilasters are taken as such, and the thickness in all cases stated. The bases and capitals of both plain and fluted pilasters are taken by themselves, as the mouldings of the pilasters.

Solid columns are taken by their height and greatest diameter; and when their mouldings are turned out of the solid, the diameter is taken at the base. The shafts of built columns are taken superficially by the whole height, and by the girt of the greatest diameter, and allowed two measures. When columns are fluted and reeded, they are taken as such; and if reeds are planted in, they are taken lineally. The bases and capitals are measured as mouldings, and the circular part only allowed double measure.

Facings, skirtings, base-plinths, and door-stops, under 8 inches broad, are reduced to 4 inches broad; and all above 8 inches broad are taken as plain linings.

The stanchel part of railing is taken by the yard, stating the size of the stanchels and the distance between them; the posts and rails are reduced to 4 inches broad. The posts of the rail are included in the surface-measure.

The Chinese part of railing is measured by the square yard, as such, the posts reduced to 3 inches square, and the rails to 4 inches broad, stating the thickness.

The square steps of timber stairs are taken by their length and by girting over the step and breast, allowing an inch of cover to each. The wheel steps are taken at their extreme length, and by girting at the mean breadth, allowing 3 inches on each step for cutting. Spring-boards and brackets are taken by the square foot, specifying their thickness.

Stair hand-rails are taken by the lineal foot, stating the quality. Circular parts are double measure, twist and circle three measures, and the measure taken round the scroll.

1. What is the value of a sash-window which measures 6 feet 10 inches by 3 feet 8 inches, at 2s. per square foot?

Ans. £2, 10s. $1\frac{1}{3}$d.

2. How many square yards of roofing and sarking are in a house 60 feet long from skew to skew, and each side of the

roof 22 feet, allowing 9 inches for the breadth of the wall-plate; and what is the value of it, at 9s. 6d. per square yard?

Two sides 45 feet 6 inches.
Length 60 0

9)2730(303$\frac{1}{3}$ square yards at 9s. 6d. = £144, 1s. 8d.

3. How many yards of flooring in a house of three stories, 56 feet by 28 feet within the walls, deducting the vacancy for the stair, 13 feet by 8 feet; and what is the value, at 5s. 6d. per square yard? Ans. £134, 4s.

4. How much wainscoting in a room 25 feet by 18 feet and 14 feet 3 inches high when girt over the mouldings, allowing a door 7 feet 2 inches by 3 feet 4 inches, 2 windows with shutters, each 5 feet 8 inches by 3 feet 6 inches, and a chimney 6 feet 4 inches by 5 feet 6 inches; the doors and shutters being charged work and half-work?

Ans. 135 yards 7 feet 5$\frac{1}{3}$ inches.

5. A partition is 173 feet 10 inches in length, and 10 feet 7 inches in height. How many squares are in it?

Ans. 18·397361 squares.

6. How many yards of flooring and joisting in a house of 3 floors, 48 feet by 27 within walls, allowing 9 inches for the rests of the joists, and deducting from each floor the vacancy for the stair, 12 feet by 8 feet 3 inches; and what is the expense of the materials and workmanship, the joisting and flooring at 7s. 6d. per yard, and the naked joisting at 3s. 6d. per yard? Ans. £153, 16s. 6d.

PLASTER WORK.

Plain plaster work is measured by the square yard, stating the number of coats and the quality of the finishings.

Upright circular walls, soffits of arches, coves, &c. are allowed double measure. Domes and tops of niches are allowed three measures. When new and old plaster are joined, an allowance is made of one foot for splicing; and when mouldings are put on plain plaster, to form pannels, the whole wall is taken as plain plaster, and the mouldings are taken again by the lineal foot.

Where stiles are raised, the general superficies of the wall is measured as pannelled plaster. The stiles and mouldings are taken by the lineal foot, stating the breadth.

These rules apply to ceilings as well as walls, and to mouldings, whether plain or enriched.

All circular mouldings on domes are double measure. Pannelled soffits of arches, and pannelled scontions of stair-windows are taken by girting over the mouldings both ways; and if at or above 12 inches broad, they are estimated by the square foot; but if under 12 inches, by the lineal foot, stating the breadth.

Architraves of arches are taken as other mouldings.

Plain cornices, at or above 12 inches in girt, are taken by the square foot, and all under that by the lineal foot.

Enriched cornices are measured in the same way, stating the number and nature of the enrichments; and for all mitres in a room, &c. more than four, one foot is allowed for each, whether external or internal.

Plain and enriched entablatures are measured by the square foot, by girting from the ceiling down to the plain plaster of the walls; and the number and quality of the enrichments are stated.

Entablatures on the bottom of coves are measured on the upper bed, as far as the mould goes back, and down to the plain plaster.

If the ornaments and mouldings on a ceiling do not exceed 12 inches in their distance from each other, the whole ceiling is taken by the superficial foot, as an ornamented one; but when their distance exceeds 12 inches, the mouldings and margins are taken in the same way as pannelled plaster.

Centre ornaments above 3 feet diameter are taken by the square foot, and all at or under that by the piece, stating the size.

Heads, trusses, and other detached ornaments, are reported by the number and size.

Plaster beads are taken as plain mouldings, and relieved corner beads by the lineal foot, as double cut.

1. How much plastering on a partition 7 feet 8 inches long and 10 feet 3 inches high, deducting a door 6 feet 3 inches by 2 feet 10 inches; and what will it cost, at 5d. per yard?

10 feet	3 inches.		6 feet	3 inches.
7	8		2	10
78	7 wall.		17	$8\frac{1}{2}$ door.
17	$8\frac{1}{2}$ door.			
9)60	$10\frac{1}{2}$ content.			

6 yards 6 feet $10\frac{1}{2}$ inches content, at 5d. is 2s. $9\frac{3}{4}$d.

2. How many square yards of plastering on the walls and ceiling of a room 30 feet long, 25 broad, and 12 high, deducting 3 windows, each 8 feet 2 inches by 5 feet, 2 doors, each

7 feet by 3 feet 6 inches, and a fireplace 4 feet 6 inches by 4 feet 10 inches, the sides of the windows behind the shutters being plastered, and measuring 8 feet 2 inches by 15 inches; and what will it cost, at $6\frac{1}{4}$d. per square yard?

Ans. 215 yards 3 feet, cost £5, 12s. $1\frac{3}{4}$d. $\frac{1}{3}$.

SLATERS' WORK.

Square roofs are girted for their deepness from the top of the ridge downwards, allowing 9 inches for the double eaves, and for the length between the skews, and 6 inches more for cover.

Chimney-stacks, and all apertures above 4 square feet of daylight, are deducted, allowing the double eaves above such openings, and also 9 inches for cutting along each side; but no deductions are made at or under 4 square feet.

Stormont and roof windows are measured according to the form of the different parts, and 9 inches by the length allowed for every cutting on peends, flanks, and skews.

Close flanks made waterproof without lead are allowed double of a common flank for cutting.

Circular work and dome roofs are double measure. Ridge stones are reported by the lineal foot.

Tile roofs are measured in the same way as slate roofs, but no allowance for double eaves, unless when slate eaves are put on, in which case 6 inches more than what is seen is allowed on the slating for cover.

The pointing of slate or tile roofs is measured as before stated, but no allowance for cutting or for eaves. The deepness of the plaster is to be added to the length of the roof.

Slate and tile roofs are estimated by the rood of 36 square yards.

1. How much slating is in a roof 46 feet long, and 18 feet from the coping to the eaves? Ans. 5 roods 11 yards 6 feet.

2. Required the content of a tile roof 42 feet 7 inches long, and 16 feet 10 inches from the ridge to the eaves; and what does it amount to, at £3, 15s. per rood?

Ans. 4 roods 15 yds. 2 ft. 7 in. 8 pts. cost £16, 11s. $10\frac{1}{4}$d.

3. Required the expense of a slate roof measuring 48 feet 6 inches in length, and 24 feet from ridge to eaves, breadth of the wall-plate 9 inches, reckoning the roofing and sarking at 7s. per square yard, and the slating, including slates, at £5, 8s. per rood. Ans. £133, 7s. 6d.

PAINTERS' WORK.

PLAIN painting is measured wherever the brush touches, and estimated by the square yard, stating the colour and quality, whether oil or size, and the number of coats.

Party-coloured work is measured first as plain work, and then the stiles and mouldings are taken and estimated by the lineal foot, according to the number of different colours; and this rule applies to skifting and mouldings of a room, when different colours form the general body of the work.

An allowance of 6 inches for each enrichment in cornices is added to the girt, when enriched cornices are picked in; and if at or above one foot of girt, they are taken by the superficial foot; all under that girt by the lineal foot. In both cases, the number of enrichments are to be stated, besides being included along with the plain work with which they may class.

Ornamented ceilings are measured in the same way as plaster work.

Mock mouldings in passages, staircases, &c. are reported by the lineal foot. Outsides of windows are allowed one-fourth more than the net daylight.

Stanchel-railing, at or under 6 inches in the open, is allowed double measure; above 6 and under 9 inches, measure and half; from 9 to 12 inches, one and one-fourth; and all above that, single measure. Stanchels put into windows are taken by including one of the side spaces between the stanchel and the rybats.

Ornamented railing on stairs is allowed double measure, and figures of every description are reported by number.

1. How much painting on a wall 14 feet by $9\frac{1}{2}$ feet, deducting the chimney, 4 feet 6 inches by 3 feet 10 inches; and what does it come to, at 10d. per square yard?

Ans. Content 12 yards $7\frac{3}{4}$ feet, value 10s. $8\frac{1}{2}$d.

2. A room is 20 feet long, 14 feet 6 inches broad, and 10 feet 4 inches high. How much painting is in it, deducting a fireplace 4 feet 4 inches by 4 feet, and 2 windows, each 6 feet by 3 feet 2 inches? Ans. 73 yards $0\frac{2}{3}$ foot.

3. Required the expense of painting a room 28 feet long and 20 broad, the girt of the wainscoting or *dado-work* round the bottom of the room 2 feet 10 inches by 84 feet; the height from the wainscoting to the ceiling 7 feet 10 inches; 3 windows, each 7 feet 10 inches by 4 feet 9 inches; 2 doors, and 2 presses, each 7 feet 6 inches by 4 feet; and a fireplace 4 feet 9 inches by 5 feet. The wood work is painted in oil,

the window-shutters and doors on both sides, at 9d. per square yard; the walls with size at 3d., and the ceiling is white-washed at $1\frac{1}{2}$d. per yard. Ans. £4, 1s. $11\frac{3}{4}$d. $\frac{2}{3}$.

GLAZIERS' WORK.

GLASS is measured by the superficial foot, stating the quality. Every pane is measured at the extreme points, including the back-check of the astragal.

1. A window is 5 feet 4 inches by 3 feet 2 inches of day-light. What does the glazing amount to at 14d. per square foot? Ans. Content $16\frac{8}{9}$ feet, value 19s. $8\frac{1}{2}$d.

2. An oval window is 4 feet 3 inches by 2 feet 5 inches. Required the expense of glazing it, at 1s. 3d. per square foot. Ans. Content $10\frac{1}{4}\frac{3}{8}$ feet, value 12s. 10d.

3. Required the expense of glazing the windows of a house of three stories, at 1s. 4d. per square foot, the common breadth of the windows being 3 feet 10 inches; the height of the lower tier 7 feet 8 inches, of the second 6 feet 10 inches; and of the highest 5 feet 3 inches; 4 windows in each tier.
Ans. £20, 3s. $9\frac{1}{3}$d.

PLUMBERS' WORK.

PLUMBERS' WORK is generally done by the pound or hundred-weight; but the laying down of lead is done by the day.

Sheet-lead used in roofing, &c. weighs from 7 to 12 lb. per square foot. Leaden pipes of $\frac{3}{4}$ inch bore weigh 10 lb.; of 1 inch bore, 12 lb.; of $1\frac{1}{4}$ inch bore, 16 lb.; of $1\frac{1}{2}$ inch bore, 18 lb.; of $1\frac{3}{4}$ inch bore, 21 lb.; and of 2 inches bore, 24 lb. per yard, in length.

1. Required the expense of a leaden pipe of $1\frac{1}{4}$ inch bore, and 72 feet long, at $3\frac{1}{4}$d. per lb. Ans. £5, 4s.

2. Required the expense of lining a water-cistern 2 feet 10 inches long, 2 feet 6 inches deep, and 2 feet broad, with sheet-lead of 10 lb. to the square foot, at £1, 18s. 9d. per cwt.
Ans. £5, 3s. $2\frac{1}{2}$d. $\frac{13}{28}$.

3. The platform on the roof of a square building measures 40 feet square, and is covered with lead of 9 lb. to the square foot; the hips are each 16 feet 6 inches long, and covered to the breadth of 18 inches with lead of 10 lb. to the square foot; the water-pipe is of 1 inch bore and 48 feet long, and the soil-pipe is of 2 inches bore and 30 feet long; the water-cistern is 3 feet 6 inches long, 2 feet 6 inches deep, and 3 feet

wide, and lined with lead of 11 lb. to the square foot. Required the expense of the whole, the sheet-lead being rated at £1, 11s. 6d. per cwt., and the pipes at 4¾d. per lb.

Ans. £231, 12s. 5½d. ½.

PAVIERS' WORK.

Causewaying is measured by the rood or yard, stating whether rubble or coursed work. One foot by the length is added as an allowance for every channel, and 6 inches by the length for cutting on coursed work, and for warpings.

Hewn pavement is measured by the square foot, stating the quality; and, if grooved pavement, the grooves are added to the surface measure.

The hollow part of gutters cut in pavement is taken over again; and sinks are taken two times, after being included in the surface-measure.

1. A court-yard is 50 feet long by 40 feet 6 inches broad. What will the paving of it amount to, at 3s. 7½d. per square yard? Ans. £40, 15s. 7½d.

2. What will be the expense of paving a square court, the length of the side being 150 feet? The outside, to the breadth of 10 feet, is paved with Arbroath pavement at 3s. per square yard, and the rest is done with common pavement at 1s. 9d. per yard. Ans. £257, 12s. 9⅓d.

3. A hexagonal space, the outside of which to the breadth of 12 feet, in a line from the corner to the centre, is to be paved with Arbroath pavement at 2s. 10½d. per yard; the remainder, deducting a circular garden in the centre, of 300 feet diameter, is to be done with common pavement at 1s. 8¾d. per yard. Required the amount of the expense, supposing the length of the side 250 feet. Ans. £977, 14s. 1d

OF VAULTS.

Vaults are formed by arches springing from opposite walls, and meeting in a line at the top.

Prob. I. To find the surface of a vault.

Rule. Apply a line close to the arch, from one side to the other, to get the girt, and multiply it by the length of the vault to get the surface; and this, multiplied by the thickness of the arch, will give the solid content of the arch.

1. Required the surface of a vault 106 feet long, and the girt of the arch $42\frac{3}{8}$ feet. Ans. $499{\cdot}3\dot{7}$ yards.

2. Required the surface of a vault 56 feet long, the girt of the arch 36 feet 4 inches; and also the solidity of the arch, its thickness being 2 feet.

Ans. $226\frac{8}{27}$ yards surface, 150 yards $19\frac{1}{3}$ feet solidity.

3. Required the surface of a vaulted roof, the length being 125 feet, and the girt 36 feet. Ans. 500 square yards surface.

Prob. II. To find the concavity of a vault.

Rule. Find the area of one of its ends according to its form, whether circular, elliptical, or Gothic, and multiply it by the length of the vault.

1. Required the content of a semicircular vault, the span being 30 feet, and the length 150 feet.

Ans. 53014·5 cubic feet.

2. Required the content of an oval vault, the span being 30 feet, the height 12, and the length 60 feet.

Ans. 16964·64 cubic feet.

3. Required the vacuity of a Gothic vault 20 feet long, the span 50 feet, the chord of each of the arches 50 feet, and the versed sine of the arch 15 feet. Ans. 43024·215 cubic feet.

OF GROINS.

Groins are formed by the intersection of vaults with one another.

Prob. I. To find the surface of a groin.

Rule I. Divide the area of the base by 7, and add the quotient to the dividend: the sum will be the area.

Note. This rule is correct only when the groin is a semicircle.

1. Required the surface of a groin raised upon a square, of which each side is 14 feet. Ans. 224 square feet.

2. Required the surface of a groin raised upon a rectangular base, of which the sides are 14 and 18 feet.

Ans. 288 square feet.

3. Required the surface of a circular groin-arch raised on a square base, each side 20 feet. Ans. $457\frac{1}{7}$ square feet.

Rule II. Multiply the square of the base by 1·1416 for the surface of the groin-arch, and add to the product twice the product of the diameter of the four semicircular spaces between the piers into their breadth, and into 3·1416 for the whole surface required.

1. Required the surface of a groin-arch, 30 feet square, having 4 semicircular spaces between the piers, each 30 feet in diameter and 18 inches broad.

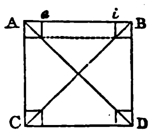
$30 \times 30 \times 1\cdot1416 = 1027\cdot44$ square feet, the groin, and $30 \times 2 \times 1\cdot5 \times 3\cdot1416 = 282\cdot744$ feet, surface of semicircular spaces; then $1027\cdot44 + 282\cdot744 = 1310\cdot184 = 1310\cdot184$ square feet, the whole surface.

2. Required the surface of a groin-arch, 33 feet square, having 4 semicircular spaces between the piers, each 33 feet in diameter and 20 inches broad. Ans. 1588·7784 sq. feet.

Prob. II. To find the solidity of the masonry in a semicircular groin-arch.

Rule I. Multiply the product of the length and breadth by the height from the springing of the arch to the top; from this product subtract the square of the inside measure, multiplied by the height within, and by $\cdot9041\dot{3}$; subtract also the four semicircles between the piers, and the last remainder will be the solid content of the arch when made up level to the crown of the arch.

1. There is a semicircular groin-arch, 15 feet high, and the opening within, *ei*, 30 feet; the arch *Ae*, or *iB*, is 18 inches thick, and the square of the arch over the piers ABCD is 33 feet in the side. Required the cubic content of the arch.



First $33^2 \times 16\cdot5 = 17968\cdot5$, and $30^2 \times 15 \times \cdot9041\dot{3} = 12205\cdot8$; also $30^2 \times \cdot7854 \times 2 \times 1\cdot5 = 2120\cdot58$; then $17968\cdot5 - (12205\cdot8 + 2120\cdot58) = 17968\cdot5 - 14326\cdot38 = 3642\cdot12$ cubic feet, the solidity.

Note. As the arch is generally of a better description of materials than the making up of the corners, it is therefore necessary sometimes to find the arch separately, which may be done by the following rule.

Rule II. Find the cubic content of a hollow cylinder, whose length and diameter over all is the same as that of the arch, and whose inside diameter is the same as the inside of the arch. Square the diameter over all, and multiply the product by the half of the same diameter; from this product subtract the square of the inside diameter, multiplied by its half, and subtract $\frac{2}{3}$ of the remainder from the content of the cylinder: the remainder is the content of the groin-arch, including the four spaces between the piers.

Taking the last example, we have $33^3 \times \cdot7854 - 33 \times 30^2 \times \cdot7854 = 4898\cdot5398$ the content of the cylinder; then $33^2 \times 16\cdot5 - 30^2 \times 15 = 4468\cdot5$, $\frac{2}{3}$ of which is 2979; hence $4898\cdot5398 - 2979 = 1919\cdot5398 =$ cubic feet, content of

the arch, which, taken from the content found by Rule I. or $3642\cdot12 - 1919\cdot5398 = 1722\cdot5802$ cubic feet, the filling up of the corners.

2. There is a semicircular groin-arch, 25 feet opening within, and $12\frac{1}{2}$ feet high, the arch is 18 inches thick, and the square of the arch over the piers is 28 feet. Required the cubic content of the arch. Ans. 1387·6 cubic feet.

PROB. III. To find the vacuity of a groin.

RULE. Multiply the area of the base by the height, and from the product subtract $\frac{1}{10}$ of it: the remainder will be the solidity.

NOTE 1. Instead of subtracting $\frac{1}{10}$ of the product, it may be multiplied by ·9, or by ·904.

NOTE 2. This rule is correct only when the groin is a semicircle.

1. Required the vacuity of a circular groin upon a square base, of which the side is 14 feet, and its height 7 feet.

Ans. $14^2 \times 7 - 14^2 \times \cdot9 = 1234\cdot8$ cubic feet.

2. Required the vacuity formed by an elliptical groin, the side of its square base being 28 feet, and its height 9 feet.

Ans. 6350·4 cubic feet.

3. Required the vacuity of an elliptical groin upon a rectangular base 20 feet by 30, and the height 12 feet.

Ans. 6480 cubic feet.

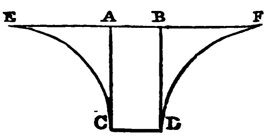
OF BRIDGES.

PROB. To measure the spandril walls of a bridge when they are thicker at the bottom than the top.

RULE. Find the areas of each of the pieces separately, multiply each by its mean thickness, and the sum of the products is the content.

The mean thickness of the centre part over the pier is half the sum of the thickness at the top and at the bottom, and the mean or average thickness of the circular part is found by adding to the top thickness $\frac{1}{4}$ of the difference between the thickness at the bottom and that at the top.

1. The wall ABCD over the pier of a bridge is 30 feet high and 10 feet broad, the thickness at the bottom is 2 feet, and that at the top 1 foot 6 inches; the two quarter circles EAC, and BFD are the same thickness, and each 30 feet by 30 feet. Required the content of the masonry in cubic feet.



First $\frac{1}{2}(2+1\cdot5) = 1\cdot75$ mean thickness of the wall over the pier, and $1\cdot5 + \frac{1}{4}(2 - 1\cdot5) = 1\cdot5 + \cdot125 = 1\cdot625$ foot mean thickness of the quarter circles; then $30 \times 10 \times 1\cdot75 = 525$ cubic feet over the pier, and $(30^2 - 30^2 \times \cdot7854) \times 2 = 193\cdot14 \times 2 = 386\cdot28$ the area of both sides of the circular part, and $386\cdot28 \times 1\cdot625 = 627\cdot705$; then $525 + 627\cdot705 = 1152\cdot705$ cubic feet the content.

2. The wall over the pier of a bridge is 40 feet high and 12 feet broad, the thickness at the bottom 2 feet 6 inches, and at the top 1 foot 9 inches; the two quarter circles are the same thickness, and each 40 feet by 40 feet. Required the content of the masonry in cubic feet.

Ans. 2350·52 cubic feet.

OF DOMES.

A Dome is formed by arches springing from a circular or polygonal base, and meeting in a point at the top.

Prob. I. To find the surface of a spherical dome.

Rule I. Multiply twice the area of the base by the height; and the product, divided by the radius of the base, will give the surface.

Note. This rule is accurate only when the dome is a semicircle.

Rule II. Multiply the circumference of the great circle of the dome (or that of which double the radius is the diameter) by the perpendicular height, the product is the curve surface.

1. The chord line or diameter at the base of a dome is 80 feet, and the perpendicular height or versed sine is 30 feet. Required the curve surface.

First $\left(\frac{80}{2}\right)^2 \div 30 = 40^2 \div 30 = 1600 \div 30 = 53\cdot\dot{3}$ and $53\cdot\dot{3} + 30 = 83\cdot\dot{3}$ feet, the diameter of the great circle of the dome, and $83\cdot\dot{3} \times 3\cdot1416 = 261\cdot8$ feet, the circumference; then $261\cdot8 \times 30 = 7854$ feet, the surface.

The area by Rule I. is only 7539·84 feet, or 314·16 feet too little.

2. Required the surface of a spherical dome upon a hexagonal base, of which the side is 10 feet.

Note. The radius of the base being equal to the height, twice the area of the base is the surface = 519·61524 square feet.

3. Required the expense of painting a spherical dome upon an octagonal base, of which the side is 20 feet, at 8d. per square yard.

Ans. £14, 6s. 1¼d.

PROB. II. To find the surface of a spherical dome which has a circular opening for a lantern or cupola at the top.

RULE. Divide the product of half the sum of the chord and the diameter of the top opening multiplied by half their difference by the perpendicular height, and add the quotient to the perpendicular height. To the square of half this sum, add the square of half the diameter of the top opening, and take the square root of the sum for the radius of the great circle of the dome. Multiply the circumference of the great circle by the perpendicular height for the curve surface of the dome.

1. A spherical dome whose chord line at the bottom is 88 feet, has a circular opening at the top, the diameter of which is 30 feet; the perpendicular height from the chord line to the diameter of the top opening is 33·42 feet. Required the curve surface. Ans. 1047 yards 5 feet 3·42 inches.

2. A dome at the bottom is 102 feet in diameter, the lantern opening is 40 feet in diameter, and the perpendicular height is 38 feet. Required the curve surface.

Ans. 1378 yards 4 feet $10\frac{2}{3}$ inches.

3. There is a spherical dome, covered with lead, whose chord line at the bottom, across the base, is 68 feet; at the top is a circular opening for a skylight 20 feet in diameter; the perpendicular height from the top opening to the base chord is 20 feet; there are 28 battens or rolls for the lead, each making an additional 6 inches: how many square feet of lead is on the dome, including the 28 rolls, what is its weight at $7\frac{1}{2}$ lbs. per square foot, and what is the cost at 1s. 9d. per foot?

Ans. 5195·182 square feet, which weighs 347 cwt. 3 qrs. 15 lb. 13 oz. ·13 drams, and costs £454, 11s. $6\frac{3}{4}$d.

4. There is a staircase 13 feet 6 inches square, on which is a spherical dome whose diameter from angle to angle of the staircase is 18 feet, the perpendicular height from the base to the opening of the circular skylight is 6 feet 9 inches; there are four semicircular spaces to be deducted, whose chord line is 12 feet, the depth is 27 inches, or half the difference between the diameter of the dome and the square side of the staircase. Required the superficial content of the four spherical angle spaces left. Ans. 14 square yards 1 foot $2\frac{3}{4}$ inches.

PROB. III. To find the surface of an elliptical dome.

RULE. Divide the difference between the squares of the diameters by the square of the less diameter, if for an oblate;

but by the square of the greater, if for an oblong spheroid, and call the quotient x.

Add $\frac{1}{6}$ of x to unity, if for an oblate; but subtract it from unity if for an oblong spheroid, and take $\frac{2}{3}$ of this sum or difference, which call y.

From this sum or difference, subtract $\frac{1}{40}$ of x^2 multiply the remainder by $\frac{8}{27}$, and *retain* the product. Add $\frac{1}{8}$ of x to unity if for the oblate; but subtract it from unity if for the oblong spheroid; from the square root of the sum or difference take the number represented by y, and subtract the remainder from the product which was *retained*. Multiply this remainder by both the diameters, by 3·1416, and by $3\frac{3}{7}$, the product will give the curve surface of either spheroid.

1. An elliptical dome, 20 feet diameter at the base, and 20 feet high, being half an oblong spheroid, is to be covered with lead at 7 lb. per square foot, having 20 rolls or battens, each 6 inches additional to the surface. Required the superficial content of the lead, its weight, and also the whole expense at 1s. 8d. per square foot.

Ans. Content 1311 square feet 10 in. 5 pts. Weight 81 cwt. 3 qrs. 27 lb. 1 oz. $11\frac{3}{4}$ drams, and expense £109, 6s. $5\frac{1}{4}$d.

2. There is an elliptical dome (half of an oblate spheroid), the diameter at the base is 60 feet, and the perpendicular height 24 feet, how many yards of plaster does it contain?

Ans. 546 yards 4 feet 8 inches.

PROB. IV. To find the vacuity of a spherical dome.

Multiply the area of the base by two-thirds of the height.

NOTE. This rule is true only when the dome is a semicircle.

1. Required the content of a spherical dome, the diameter of its circular base being 30 feet.

Ans. $30^2 \times {\cdot}7854 \times \frac{2}{3} \times 15 = 7068{\cdot}6$ cubic feet.

2. Required the solid content of an octagonal dome, of which the height is 21 feet, and each side of the base 20 feet.

Ans. 27039·1912 cubic feet.

3. Required the solid content of a dome upon a nonagonal base, of which the side is 12 feet, and the height 30 feet.

Ans. 17803·653696 cubic feet.

OF SALOONS.

SALOONS are formed by arches connecting the side-walls of a building with a ceiling or platform in the middle.

PROB. I. To find the surface of a saloon.

RULE. Apply a line close to the arch, across the surface. from the side-wall to the platform, for its breadth, then measure along the middle of it quite round the room for its length, and multiply one of these by the other, to get the surface.

1. The girt across the face of a saloon is 4 feet, and the mean length round the room is 108 feet. Required the surface. Ans. 432 square feet.

2. The girt across the face of a saloon is 7 feet 10 inches, and the mean length round the room 140 feet. What will the plastering of it cost, at $6\frac{3}{4}$d. per square yard, and the painting in oil, at 15d. per square yard?

Ans. £3, 8s. $6\frac{1}{2}$d. plastering; £7, 12s. $3\frac{3}{4}$d. $\frac{1}{9}$ painting.

3. The mean length of a saloon is 127 feet 6 inches, and the breadth across the face of the saloon 6 feet. What will the size-painting of it cost, at $4\frac{1}{4}$d. per square yard?

Ans. £1, 10s. $1\frac{1}{4}$d.

PROB. II. To find the area of the concave part of a quadrantal saloon.

RULE I. Multiply 3·1416 by half the radius of the arch for the girt across the arch. Multiply the girt across the arch by the length round the ceiling at the top of the arch, and to this product add the areas of two circles, whose diameters are twice the radius of the arch for a circular saloon; but when the saloon is square or oblong, add the areas of two squares, whose sides are double the radius, and the sum will be the area required.

RULE II. Find the length round the room at the top of the arch, and also at the bottom of the arch. To half the sum of these lengths, add the product of the radius into 1·091 when the room is square or oblong; but into ·8584 when the room is circular; then multiply the sum by the girt across the arch for the area required.

1. The diameter of a circular room is 60 feet, over which springs a quadrantal arch of 5 feet radius. Required the curve-surface of the arch.

By Rule I. $60 - 10 = 50$ feet, the diameter at the flat ceiling; $50 \times 3{\cdot}1416 = 157{\cdot}08$, the circumference at the ceiling; and $3{\cdot}1416 \times 10 \div 4 = 7{\cdot}854$ feet, the girt across the curve. Now, $157{\cdot}08 \times 7{\cdot}854 = 1233{\cdot}70632$ feet, and $10^2 \times 2 \times {\cdot}7854 = 157{\cdot}08$ twice the area of a circle whose diameter is double the radius; hence $1233{\cdot}70632 + 157{\cdot}08 = 1390{\cdot}78632$ square feet, the curve-surface required.

By Rule II. $\frac{1}{2}$ $(60 \times 3{\cdot}1416 + 50 \times 3{\cdot}1416) = \frac{1}{2}$ $(188{\cdot}496 + 157{\cdot}08) = 172{\cdot}788$ half the sum of the lengths, and $5 \times {\cdot}8584 = 4{\cdot}292$; then $172{\cdot}788 + 4{\cdot}292 \times 7{\cdot}854 = 177{\cdot}08 \times 7{\cdot}854 = 1390{\cdot}78632$ square feet, the same as before.

2. There is a room 80 feet long and 60 feet wide, over which springs a quadrantal arch of 6 feet radius. Required the curve-surface of the arch.

By Rule I. $80 - 12 = 68$ feet, the length at the flat ceiling; $60 - 12 = 48$ feet, the breadth at the flat ceiling; $68 + 48 \times 2 = 232$ feet, the girt at the ceiling; and $3{\cdot}1416 \times 12 \div 4 = 9{\cdot}4248$, the girt across the curve. Now, $232 \times 9{\cdot}4248 = 2186{\cdot}5536$ feet, and $12^2 \times 2 = 288$ feet, twice the square of double the radius; hence $2186{\cdot}5536 + 288 = 2474{\cdot}5536$ square feet, the curve-surface required.

By Rule II. $\frac{1}{2}$ $(232 + 280) = 256$ half the sum of the lengths, and $1{\cdot}091 \times 6 = 6{\cdot}546$; then $(256 + 6{\cdot}546) \times 9{\cdot}4248 = 262{\cdot}546 \times 9{\cdot}4248 = 2474{\cdot}4435$ square feet, nearly the same as before.

3. There is a saloon 60 feet square, having a quadrantal arch of 5 feet radius. Required the concave surface of the saloon.

Ans. 1770·8 square feet by Rule I., and 1770·72357 square feet by Rule II.

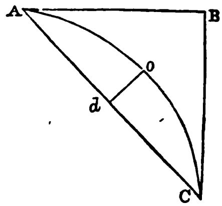
4. There is a circular saloon, 40 feet in diameter, having a quadrantal arch whose radius is 5 feet. Required the curve surface of the saloon. Ans. 897·303792 square feet.

PROB. III. To find the vacuity of a saloon.

RULE I. Multiply the difference between the area of the triangle and the area of the segment by the length of the room, and subtract the product from the cubic content of the room, the remainder is the vacuity of the saloon.

NOTE. Multiply $\frac{2}{3}$ of the chord of the segment by the versed sine, and to this product add the quotient of the cube of the versed sine, divided by twice the chord; the sum will give the area of the segment.

1. Suppose the perpendicular height of a saloon to be 38·4 inches, the horizontal distance from the platform to the side-wall 37·9 inches, the chord of the arch 54 inches, and the distance of its middle point from the arch 9 inches, the chord of half the arch 28·44 inches, and the compass round the middle of the saloon 50 feet. Required the vacuity.



First $37{\cdot}9 \times 19{\cdot}2 \div 144 = 727{\cdot}68 \div 144 = 5{\cdot}05\dot{3}$ feet, area of the triangle ABC; then $(4{\cdot}5 \times \frac{2}{3} \times {\cdot}75) + ({\cdot}75^3 \div 9) = 2{\cdot}25 + ({\cdot}421875 \div 9) = 2{\cdot}25 + {\cdot}046875 = 2{\cdot}296875$ feet, area of the segment A*o*C. Now, $5{\cdot}05\dot{3} - 2{\cdot}296875 \times 50 = 2{\cdot}756458\dot{3} \times 50 = 137{\cdot}82291\dot{6}$ cubic feet, content occupied by the saloon, which, taken from the cubic content of the room, will leave the vacuity of the saloon.

Rule II. When the size of the room, &c. is given.

1. Multiply the height of the arc by its projection by $\frac{1}{4}$ of the perimeter of the ceiling, and by $3{\cdot}1416$, for the first product.

2. From a side or diameter of the room take a like side or diameter of the ceiling, and multiply the square of the remainder by $\frac{2}{3}$ of the height, and by 1 if the room is square or rectangular, but by $\cdot7854$ if the room is circular; or, if the room is a regular polygon, multiply by the area of that polygon whose side is unity for the second product.

3. Multiply the area of the flat ceiling by the height of the arch, and add this and the two former products together for the vacuity required.

1. What is the vacuity of a saloon with a circular quadrantal arch, of 2 feet radius, springing over a rectangular room 20 feet long and 16 feet wide, the projection on each side being 2 feet?

Here the flat part of the ceiling is 16 feet by 12, hence the perimeter $= 56$.

First $3{\cdot}1416 \times 2 \times 2 \times 14 = 175{\cdot}9296$ first product; then $(20 - 16)^2 \times 2 \times \frac{2}{3} = 4^2 \times 2 \times \frac{2}{3} = 21{\cdot}\dot{3}$ feet, second product; and $16 \times 12 \times 2 = 192 \times 2 = 384$. Now, $175{\cdot}9296 + 21{\cdot}\dot{3} + 384 = 581{\cdot}2629\dot{3}$ cubic feet, the vacuity.

2. A circular building of 40 feet diameter, and 25 feet high to the ceiling, is covered with a saloon whose circular arch is 5 feet radius. Required the capacity of the room in cubic feet. Ans. $30779{\cdot}45948$ cubic feet.

Rule III. From the cubic content of the whole void space before the saloon is formed, deduct the cubic content of the space occupied by the arched bracket, the remainder is the cubic content of the vacuity required.

NOTE 1. The cubic content of the space formed by the bracket is found by multiplying the area of a cross section by the length round the room.

NOTE 2. The area of a quadrantal bracket is found by deducting the product of the square of the radius into ·7854 from the square of the radius.

NOTE 3. The length of the space occupied by the bracket is found by taking the girt round the room at the bottom, and also round the ceiling at the top of the arch, and adding to the half of their sum the product of the radius into 1·738173 when the room is circular; but into 2·2132 when the room is square or oblong.

1. A circular room 40 feet in diameter, and 25 feet high, is made into a saloon with a quadrantal arch of 5 feet radius. Required the vacuity of the room after the saloon is finished.

First $40 \times 40 \times \cdot 7854 \times 25 = 31416$ cubic feet, whole vacuity before the saloon is formed; then $\frac{1}{2}(40 + 30) = 35$ mean diameter, and $35 \times 3\cdot 1416 = 109\cdot 956$, and $109\cdot 956 + (1\cdot 738173 \times 5) = 109\cdot 956 + 8\cdot 690865 = 118\cdot 646865 =$ the length of the bracket; also $(5 \times 5) - (5 \times 5 \times \cdot 7854) = 25 - 19\cdot 635 = 5\cdot 365$ area of cross section of the bracket; consequently $31416 - (118\cdot 646865 \times 5\cdot 365) = 31416 - 636\cdot 5404 = 30779\cdot 4596$ cubic feet, the vacuity after the saloon is formed.

2. There is a saloon, 20 feet long, 16 feet wide, and 18 feet high, with a quadrantal arch of 2 feet radius. Required the whole vacuity. Ans. 5701·2628 cubic feet.

PROMISCUOUS QUESTIONS.

1. How many stones of a rectangular form, each 3 feet by $2\frac{1}{2}$ feet, will pave a road 40 yards long, and 6 yards broad?
Ans. 288 stones.

2. How many panes of glass, each 18 inches by 14 inches, will be required for 22 windows, each 5 feet by 3 feet 6 inches?
Ans. 220 panes.

3. What is the excess of a floor, 50 feet long by 30 broad, above two others, each of half its dimensions?
Ans. 750 square feet.

4. How much must be cut off from a board 26 inches broad, to contain $1\frac{1}{2}$ square yards? Ans. 6 feet $2\frac{10}{13}$ inches.

5. The ceiling of a room 28 feet broad, contains 114 square yards 6 feet. What is the length of the room?
Ans. $36\frac{6}{7}$ feet.

6. Along one side of a court 47 feet 9 inches square, there is a footpath 4 feet broad. What will be the expense of laying the rest of it with stones, at 6d. per square yard?
Ans. £5, 16s. $0\frac{3}{4}$d.

7. A room is 60 feet in circuit and 12 feet high. How much paper, 2 feet wide, will line it, deducting the door, 8 feet by 4 feet, and 3 windows, each 5 feet by $3\frac{1}{2}$ feet, and the chimney 4 feet square? Ans. $103\frac{1}{4}$ yards.

8. The base of a right-angled triangle is 300 feet, and the sum of the other two sides is 1000 feet. What are their lengths? Ans. 545 and 455 feet.

9. A roof which is 24 feet 8 inches by 14 feet 6 inches, is to be covered with lead, at 8 lbs. to the square foot. Required the expense, at 2 guineas per cwt. Ans. £53, 13s.

10. How many square feet of deal will be required to make a rectangular chest of which the length is to be $3\frac{1}{2}$ feet, the breadth 2 feet, and the depth 20 inches?
Ans. $32\frac{1}{3}$ square feet.

11. A beam is $8\frac{1}{2}$ inches deep and $3\frac{1}{2}$ feet broad. Re-

quired the depth of another twice as large, which is $4\frac{3}{4}$ inches broad. Ans. 12·5263 inches deep.

12. The four sides of a trapezium are, 13, 13·4, 24, and 18 feet, and the two first contain a right angle. Required the area. Ans. 253·38 square feet.

13. What will be the expense of paving a semicircular plot, of which the diameter is 14·8 feet, at 2s. 4d. per square foot? Ans. £10, 0s. $8\frac{1}{2}$d.

14. The wheels of a chaise, each 4 feet high, in turning within a ring, moved so that the outer wheel made two turns while the inner made one, and their distance from one another was 5 feet. What were the circumferences of the tracks described by them?

Ans. Outer, 62·8318 feet. Inner, 31·4159 feet.

15. A circular pond occupies half an acre. What was the length of the cord which struck the circle? Ans. $27\frac{3}{4}$ yards.

16. A right-angled triangle has its base 16, and its perpendicular 12, and a triangle is cut off from it by a line parallel to the base, of which the area is 24. What are the lengths of the sides of that triangle? Ans. 8, 6, and 10.

17. An ellipse is surrounded by a wall 14 inches thick, its axes are 840 links and 612 links. Required the quantity of ground enclosed, and the quantity occupied by the walls.

Ans. 4 acres 6 perches enclosed, and 1760·4933 square feet area of the space occupied by the wall.

18. What is the length of the side of an equilateral triangle, which cost as much for paving the area of it, at 8d. per square foot, as for palisading its 3 sides at a guinea per lineal yard? Ans. 72·746 feet.

19. How long must be the tether of a horse which will allow him to graze quite round an acre of ground?

Ans. $39\frac{1}{4}$ yards.

20. How many 3 inch cubes may be cut out of a 12 inch cube? Ans. 64 cubes.

21. What will be the expense of painting a conical spire, of which the height is 118 feet, and the circumference of the base 64 feet, at 8d. per square yard? Ans. £14, 0s. 8·96d.

22. The diameter of a standard bushel is $18\frac{1}{2}$ inches, and its depth 8 inches. What must be the diameter of that bushel which is $7\frac{1}{2}$ inches deep? Ans. 19·10672 inches.

23. What will be the expense of gilding a globe, of which the diameter is 6 feet, at $3\frac{1}{2}$d. per square inch?

Ans. £237, 10s. 1·19d.

24. A farmer borrowed a cubical piece of hay, which measured 6 feet every way, and he repaid two cubical pieces, of which the sides were 3 feet each. What part of the quantity borrowed has he returned? Ans. The fourth part only.

25. A person wants a cylindrical vessel 3 feet deep, which shall hold twice as much as another 28 inches deep, and 46 inches in diameter. What must be the diameter of the required vessel? Ans. 57·373 inches.

26. What will be the diameter of a globe, of which the superficial and solid contents are both expressed by the same number? Ans. 6.

27. A sack $22\frac{1}{2}$ inches broad when empty, will contain 3 bushels of corn when filled. What will another sack contain, which is twice its breadth, and of the same length?

Ans. 12 bushels.

28. A cable 3 feet long, and 9 inches in circuit, weighs 22 lbs. What will be the weight of a fathom of that cable, of which the circumference is a foot? Ans. $78\frac{2}{9}$ lbs.

29. The distance between the centres of two circles, each 50 feet diameter, is 30 feet. What is the area of the space enclosed by their circumferences? Ans. 559·115 square feet.

30. What is the length of the chord which cuts off $\frac{1}{3}$ of the area from a circle, of which the diameter is 289 feet?

Ans. 278·6538 feet.

31. A sugar-loaf in form of a cone is 20 inches high; it is required to divide it equally among three persons by sections parallel to the base. What is the height of each part?

Ans. Upper 13·8672, next 3·6044, lowest 2·5284 inches.

32. A malt-kiln is $16\frac{1}{2}$ feet square. Required the side of a square kiln, which is capable of drying three times as much malt. Ans. 28·5788 feet.

33. A round cistern is 26·3 inches in diameter, and $52\frac{1}{2}$ inches deep. What should be the diameter of another of the same depth to contain twice the quantity of liquor?

Ans. 37·1938 inches.

34. How many rafters, each $2\frac{1}{2}$ inches by $1\frac{1}{2}$ inch, can be sawed out of a square log $17\frac{1}{2}$ inches by 10 inches?

Ans. $46\frac{2}{3}$ rafters.

35. How many bricks, each 9 inches long, $4\frac{1}{2}$ inches broad, and 3 inches thick, must be taken to build a wall 100 feet long, 20 feet high, and one foot thick? Ans. $28444\frac{4}{9}$ bricks.

A TABLE

OF THE

AREAS OF CIRCULAR SEGMENTS.

Height.	Area.	Height.	Area.	Height.	Area.	Height.	Area.	Height.	Area.
.001	.000042	.051	.015119	.101	.041476	.151	.074589	.201	.112624
.002	.000119	.052	.015561	.102	.042080	.152	.075306	.202	.113426
.003	.000219	.053	.016007	.103	.042687	.153	.076026	.203	.114230
.004	.000337	.054	.016457	.104	.043296	.154	.076747	.204	.115035
.005	.000470	.055	.016911	.105	.043908	.155	.077469	.205	.115842
.006	.000618	.056	.017369	.106	.044522	.156	.078194	.206	.116650
.007	.000779	.057	.017831	.107	.045139	.157	.078921	.207	.117460
.008	.000951	.058	.018296	.108	.045759	.158	.079649	.208	.118271
.009	.001135	.059	.018766	.109	.046381	.159	.080380	.209	.119084
.010	.001329	.060	.019239	.110	.047005	.160	.081112	.210	.119897
.011	.001533	.061	.019716	.111	.047632	.161	.081846	.211	.120712
.012	.001746	.062	.020196	.112	.048262	.162	.082582	.212	.121529
.013	.001969	.063	.020691	.113	.048894	.163	.083320	.213	.122347
.014	.002199	.064	.021168	.114	.049528	.164	.084059	.214	.123167
.015	.002438	.065	.021659	.115	.050165	.165	.084801	.215	.123988
.016	.002685	.066	.022154	.116	.050804	.166	.085544	.216	.124810
.017	.002940	.067	.022652	.117	.051446	.167	.086289	.217	.125634
.018	.003202	.068	.023154	.118	.052090	.168	.087036	.218	.126459
.019	.003471	.069	.023659	.119	.052736	.169	.087785	.219	.127285
.020	.003748	.070	.024168	.120	.053385	.170	.088535	.220	.128113
.021	.004031	.071	.024680	.121	.054036	.171	.089287	.221	.128942
.022	.004322	.072	.025195	.122	.054689	.172	.090041	.222	.129773
.023	.004618	.073	.025714	.123	.055345	.173	.090797	.223	.130605
.024	.004921	.074	.026236	.124	.056003	.174	.091554	.224	.131438
.025	.005230	.075	.026761	.125	.056663	.175	.092313	.225	.132272
.026	.005546	.076	.027289	.126	.057326	.176	.093074	.226	.133108
.027	.005867	.077	.027821	.127	.057991	.177	.093836	.227	.133945
.028	.006194	.078	.028356	.128	.058658	.178	.094601	.228	.134784
.029	.006527	.079	.028894	.129	.059327	.179	.095366	.229	.135624
.030	.006865	.080	.029435	.130	.059999	.180	.096134	.230	.136465
.031	.007209	.081	.029979	.131	.060672	.181	.096904	.231	.137307
.032	.007558	.082	.030526	.132	.061348	.182	.097674	.232	.138150
.033	.007913	.083	.031076	.133	.062026	.183	.098447	.233	.138995
.034	.008273	.084	.031629	.134	.062707	.184	.099221	.234	.139841
.035	.008638	.085	.032186	.135	.063389	.185	.099997	.235	.140688
.036	.009008	.086	.032745	.136	.064074	.186	.100774	.236	.141537
.037	.009383	.087	.033307	.137	.064760	.187	.101553	.237	.142387
.038	.009763	.088	.033872	.138	.065449	.188	.102334	.238	.143238
.039	.010148	.089	.034441	.139	.066140	.189	.103116	.239	.144091
.040	.010537	.090	.035011	.140	.066833	.190	.103900	.240	.144944
.041	.010931	.091	.035585	.141	.067528	.191	.104685	.241	.145799
.042	.011330	.092	.036162	.142	.068225	.192	.105472	.242	.146655
.043	.011734	.093	.036741	.143	.068924	.193	.106261	.243	.1475[illegible]2
.044	.012142	.094	.037323	.144	.069625	.194	.107051	.244	.148371
.045	.012554	.095	.037909	.145	.070328	.195	.107842	.245	.149230
.046	.012971	.096	.038497	.146	.071033	.196	.108636	.246	.150091
.047	.013392	.097	.039087	.147	.071741	.197	.109431	.247	.150953
.048	.013818	.098	.039680	.148	.072450	.198	.110226	.248	.151816
.049	.014247	.099	.040276	.149	.073161	.199	.111025	.249	.152680
.050	.014681	.100	.040875	.150	.073874	.200	.111823	.250	.153546

Height.	Area.	Height.	Area.	Height.	Area.	Height.	Area.	Height.	Area.
.251	.154412	.301	.199085	.351	.245934	.401	.294349	.451	.343777
.252	.155280	.302	.200003	.352	.246889	.402	.295330	.452	.344772
.253	.156149	.303	.200922	.353	.247845	.403	.296311	.453	.345768
.254	.157019	.304	.201841	.354	.248801	.404	.297292	.454	.346764
.255	.157890	.305	.202761	.355	.249757	.405	.298273	.455	.347759
.256	.158762	.306	.203683	.356	.250715	.406	.299255	.456	.348755
.257	.159636	.307	.204605	.357	.251673	.407	.300238	.457	.349752
.258	.160510	.308	.205527	.358	.252631	.408	.301220	.458	.350748
.259	.161386	.309	.206451	.359	.253590	.409	.302203	.459	.351745
.260	.162263	.310	.207376	.360	.254550	.410	.303187	.460	.352742
.261	.163140	.311	.208301	.361	.255510	.411	.304171	.461	.353739
.262	.164019	.312	.209227	.362	.256471	.412	.305155	.462	.354736
.263	.164899	.313	.210154	.363	.257433	.413	.306140	.463	.355732
.264	.165780	.314	.211083	.364	.258395	.414	.307125	.464	.356730
.265	.166663	.315	.212011	.365	.259357	.415	.308110	.465	.357727
.266	.167546	.316	.212940	.366	.260320	.416	.309095	.466	.358725
.267	.168430	.317	.213871	.367	.261284	.417	.310081	.467	.359723
.268	.169316	.318	.214802	.368	.262248	.418	.311068	.468	.360721
.269	.170202	.319	.215733	.369	.263213	.419	.312054	.469	.361719
.270	.171089	.320	.216666	.370	.264178	.420	.313041	.470	.362717
.271	.171978	.321	.217599	.371	.265144	.421	.314029	.471	.363715
.272	.172867	.322	.218533	.372	.266111	.422	.315016	.472	.364713
.273	.173758	.323	.219468	.373	.267078	.423	.316004	.473	.365712
.274	.174649	.324	.220404	.374	.268045	.424	.316992	.474	.366710
.275	.175542	.325	.221341	.375	.269013	.425	.317981	.475	.367709
.276	.176435	.326	.222277	.376	.269982	.426	.318970	.476	.368708
.277	.177330	.327	.223215	.377	.270951	.427	.319959	.477	.369707
.278	.178225	.328	.224154	.378	.271920	.428	.320948	.478	.370706
.279	.179122	.329	.225093	.379	.272890	.429	.321938	.479	.371705
.280	.180019	.330	.226033	.380	.273861	.430	.322928	.480	.372704
.281	.180918	.331	.226974	.381	.274832	.431	.323918	.481	.373703
.282	.181818	.332	.227915	.382	.275803	.432	.324909	.482	.374702
.283	.182718	.333	.228858	.383	.276775	.433	.325900	.483	.375702
.284	.183619	.334	.229801	.384	.277748	.434	.326892	.484	.376702
.285	.184521	.335	.230745	.385	.278721	.435	.327882	.485	.377701
.286	.185425	.336	.231689	.386	.279694	.436	.328874	.486	.378701
.287	.186329	.337	.232634	.387	.280668	.437	.329866	.487	.379700
.288	.187234	.338	.233580	.388	.281642	.438	.330858	.488	.380700
.289	.188140	.339	.234526	.389	.282617	.439	.331850	.489	.381699
.290	.189047	.340	.235473	.390	.283592	.440	.332843	.490	.382699
.291	.189955	.341	.236421	.391	.284568	.441	.333836	.491	.383699
.292	.190864	.342	.237369	.392	.285544	.442	.334829	.492	.384699
.293	.191775	.343	.238318	.393	.286521	.443	.335822	.493	.385699
.294	.192684	.344	.239268	.394	.287498	.444	.336816	.494	.386699
.295	.193596	.345	.240218	.395	.288476	.445	.337810	.495	.387699
.296	.194509	.346	.241169	.396	.289454	.446	.338804	.496	.388699
.297	.195422	.347	.242121	.397	.290432	.447	.339798	.497	.389699
.298	.196337	.348	.243074	.398	.291411	.448	.340793	.498	.390699
.299	.197252	.349	.244026	.399	.292390	.449	.341787	.499	.391699
.300	.198168	.350	.244980	.400	.293369	.450	.342782	.500	.392699

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